GCE AS MARKING SCHEME

SUMMER 2024

AS
FURTHER MATHEMATICS
UNIT 3 FURTHER MECHANICS A
2305U30-1

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

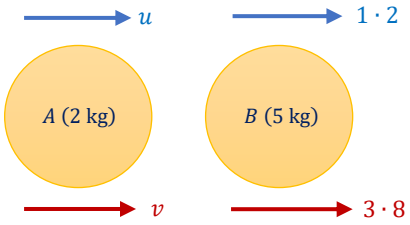
This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

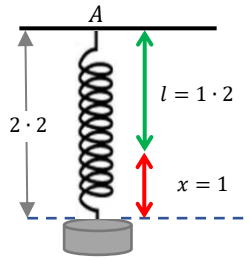
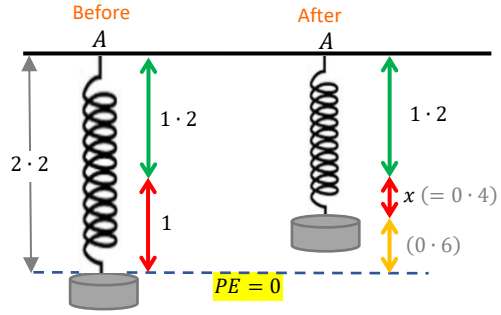
WJEC GCE AS FURTHER MATHEMATICS

UNIT 3 FURTHER MECHANICS A

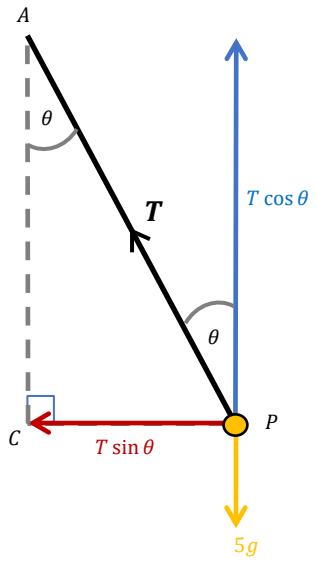
SUMMER 2024 MARK SCHEME

Q1	Solution	Mark	Notes
			$e = 0 \cdot 3$
(a)	(i) Impulse, I = Change in momentum Impulse $= (5)(3 \cdot 8 - 1 \cdot 2)$ Impulse $= 13 \quad (\text{Ns})$	M1 A1	Used cao
	(ii) $I = Ft$ $13 = F \times 0 \cdot 08$ $F = 162 \cdot 5 \quad (\text{N})$	M1 A1 **[4]**	Used to find a force FT Impulse from (i)
(b)	Conservation of momentum $(2)(u) + (5)(1 \cdot 2) = (2)(v) + (5)(3 \cdot 8)$ $2u + 6 = 2v + 19$ $u - v = 6 \cdot 5$ Restitution $3 \cdot 8 - v = -0 \cdot 3(1 \cdot 2 - u)$ $0 \cdot 3u + v = 4 \cdot 16$ $1 \cdot 3u = 10 \cdot 66$ or $1 \cdot 3v = 2 \cdot 21$ $u = 8 \cdot 2 \ (\text{ms}^{-1})$ $v = 1 \cdot 7 \ (\text{ms}^{-1})$	M1 A1 M1 A1 m1 A1 A1 **[7]**	Attempted All correct Attempted All correct, oe Eliminating one variable cao cao

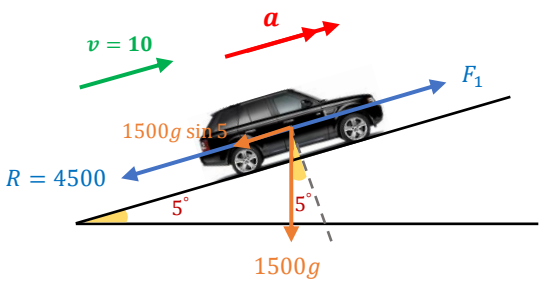
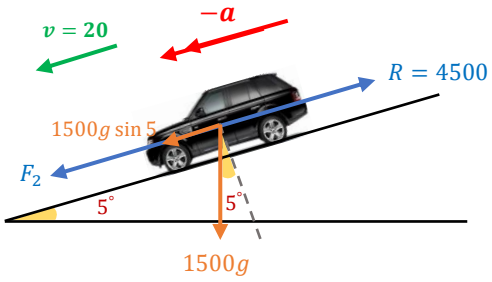
Q1	Solution	Mark	Notes
(c)	(i) $e = 0$	B1	
	(ii) Conservation of momentum	M1	Attempted for **combined** mass
	$(5)(3 \cdot 8) + (4)(0) = (5 + 4)(v)$		Zero term not required
	$19 = 9v$		
	$v = 2 \cdot 11 \ldots \quad (\text{ms}^{-1})$	A1	cao, any form, $v = \frac{19}{9} = 2 \cdot \dot{1}$
		[3]	
	Total for Question 1	**14**	

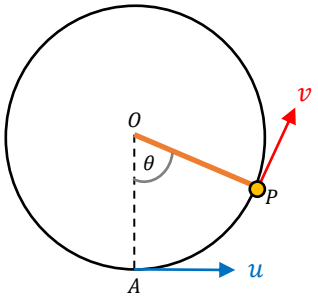
Q2	Solution	Mark	Notes
(a)			
	Using expression for $\text{EE}=\frac{\lambda x^2}{2l}$	M1	Used with $\lambda=84, l=1\cdot2$
	$\text{EE}=\frac{84(1)^2}{2(1\cdot2)}$		$x=2\cdot2-1\cdot2=1$
	$\text{EE}=35$ (J)	A1	Convincing
		[2]	
(b)			$m=4$
	Using expression for $\text{EE}=\frac{\lambda x^2}{2l}$	M1	Used with $\lambda=84, l=1\cdot2$ and $5\cdot6$
	$5\cdot6=\frac{84x^2}{2(1\cdot2)}$ $\quad(5\cdot6=35x^2)$		
	$x=\pm0\cdot4$ $\quad\therefore\quad$ $x=0\cdot4$	A1	Negative case discarded (if considered, not needed)
	Using expression for PE $(\text{PE}=\pm4gh)$	M1	Any h
	$\text{PE}=\pm\begin{cases}4g(2\cdot2-1\cdot2-0\cdot4)\\4g(0\cdot6)=2\cdot4g=23\cdot52\end{cases}$	A1	Any correct form FT derived h
	$\text{KE}=\pm\frac{1}{2}(4)v^2$ $\quad(=2v^2)$	B1	
	Conservation of energy (all energy forms)	M1	KE, EE (35 **and** $5\cdot6$), PE
	$35=5\cdot6+23\cdot52+2v^2$	A1	oe
	$v^2=\frac{147}{50}=2\cdot94$		
	$v=\frac{7\sqrt{6}}{10}=1\cdot7146\ldots(\text{ms}^{-1})$	A1	cao, any form
		[8]	
	Total for Question 2	**10**	

Q3	Solution	Mark	Notes
(a)	$\mathbf{R} = (4\mathbf{i} - 7\mathbf{j} + 9\mathbf{k}) + (5\mathbf{i} + 3\mathbf{j} - 8\mathbf{k})$ $+(-2\mathbf{i} + 6\mathbf{j} - 11\mathbf{k})$ $\mathbf{R} = 7\mathbf{i} + 2\mathbf{j} - 10\mathbf{k}$ (N)	B1 **[1]**	
(b)	Displacement vector $\mathbf{r} = (a\mathbf{i} + 7\mathbf{j} - 10\mathbf{k}) - (3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k})$ $= (a-3)\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ Work done by $\mathbf{R} = \mathbf{R}\,.\,\mathbf{r}$ $21 = (7\mathbf{i} + 2\mathbf{j} - 10\mathbf{k}).\left((a-3)\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}\right)$ $21 = (7)(a-3) + (2)(3) + (-10)(2)$ $21 = 7a - 21 + 6 - 20$ $a = 8$	M1 M1 m1 A1 **[4]**	Difference attempted cao
	Total for Question 3	**5**	

Q4	Solution	Mark	Notes
			$\tan\theta = \frac{20}{21}$ $\sin\theta = \frac{20}{29}$ $\cos\theta = \frac{21}{29}$
	Resolve vertically $T\cos\theta = 5g$	M1 A1	
	$\left(T = \frac{29}{21}\times 5g = \frac{203}{3} = \frac{145}{21}g = 67\cdot 66\ldots\right)$		
	N2L towards centre $T\sin\theta = 5a$	M1	
	$T\sin\theta = 5\frac{v^2}{r}$	m1	Using $a = \frac{v^2}{r}$
	$r = 3\sin\theta \qquad \left(r = 3\times\frac{20}{29} = \frac{60}{29} = 2\cdot 06896\ldots\right)$	B1	
	$\frac{203}{3}\times\frac{20}{29} = 5\frac{v^2}{\frac{60}{29}} \qquad \left(\frac{145}{21}g\times\frac{20}{29} = 5\frac{v^2}{\frac{60}{29}}\right)$	m1	Eliminating T
	$v^2 = \frac{560}{29} = 19\cdot 3103\ldots \qquad \left(v^2 = \frac{400}{203}g\right)$		
	$v = 4\cdot 394\ldots \quad (\text{ms}^{-1})$	A1	cao
		[7]	
	Total for Question 4	**7**	

Q5	Solution	Mark	Notes
(a)	$\mathbf{a} = \frac{\mathbf{F}}{2} = 2t\mathbf{i} - \frac{1}{2}t^{\frac{1}{2}}\mathbf{j} + 3\mathbf{k}$	B1	
	$\mathbf{v} = \int \mathbf{a}\, dt$	M1	Attempt to integrate $\mathbf{a}$
	$\mathbf{v} = \frac{1}{2} \times 2t^2\mathbf{i} - \frac{1}{3} \times \frac{1}{2}t^{\frac{3}{2}}\mathbf{j} + 3t\mathbf{k} \quad (+\mathbf{c})$ $\left(\mathbf{v} = t^2\mathbf{i} - \frac{1}{3}t^{\frac{3}{2}}\mathbf{j} + 3t\mathbf{k} \quad (+\mathbf{c})\right)$	A1	Correct integration
	At $t = 1$, $\mathbf{v} = 3\mathbf{i} - \frac{1}{3}\mathbf{j} - \mathbf{k}$ $3\mathbf{i} - \frac{1}{3}\mathbf{j} - \mathbf{k} = \mathbf{i} - \frac{1}{3}\mathbf{j} + 3\mathbf{k} + \mathbf{c}$ $\mathbf{c} = 2\mathbf{i} - 4\mathbf{k}$	m1	Used
	$\mathbf{v} = (t^2 + 2)\mathbf{i} - \frac{1}{3}t^{\frac{3}{2}}\mathbf{j} + (3t - 4)\mathbf{k} \quad (\text{ms}^{-1})$	A1 **[5]**	cao
(b)	$\mathbf{v}.(-1\mathbf{i} + 3\mathbf{k}) = 0$ $\left((t^2 + 2)\mathbf{i} - \frac{1}{3}t^{\frac{3}{2}}\mathbf{j} + (3t - 4)\mathbf{k}\right).(-1\mathbf{i} + 3\mathbf{k}) = 0$	M1	
	$(t^2 + 2)(-1) + (3t - 4)(3) = 0$	m1	FT $\mathbf{v}$ from (a)
	$t^2 - 9t + 14 = 0$		
	Solving quadratic $t = 2, 7$	m1 A1 **[4]**	cao, both values
	Total for Question 5	**9**	

Q6	Solution	Mark	Notes
	Moving **up** the slope  		Moving **down** the slope 
	$F = \frac{P}{v}$	B1	Used, si
	N2L **up** the slope	M1	All forces, dim. correct equation, F_1 and R opposing
	$F_1 - R - mg\sin\theta = ma$	A1	Correct equation
	$\frac{P}{10} - 4500 - 1500g\sin 5^\circ = 1500a$ $\frac{P}{10} - 4500 - 1281 \cdot 189 \ldots = 1500a$ $\frac{P}{10} - 5781 \cdot 189 \ldots = 1500a$	A1	Correct equation in P and a
	N2L **down** the slope	M1	All forces, dim. correct equation, F_2 and R opposing
	$F_2 - R + mg\sin\theta = -ma$	A1	Correct equation
	$\frac{P}{20} - 4500 + 1500g\sin 5^\circ = -1500a$ $\frac{P}{20} - 4500 + 1281 \cdot 189 \ldots = -1500a$ $\frac{P}{20} - 3218 \cdot 81 \ldots = -1500a$	A1	Correct equation in P and a
	Eliminating a $\frac{3P}{20} - 9000 = 0$	m1	Eliminating one variable
	$P = 60\,000$	A1	cao
	$a = 0 \cdot 145(873 \ldots)$	A1	cao
		[10]	
	Total for Question 6	**10**	

Q7	Solution	Mark	Notes
(a)			
	Conservation of energy	M1	KE and PE in dim. correct equation
	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\left(\frac{5}{7}\right)(1-\cos\theta)$ $v^2 = u^2 - 14(1-\cos\theta)$	A1 A1	KE $\pm\frac{1}{2}mu^2$ OR $\pm\frac{1}{2}mv^2$ PE $\pm mgl(1-\cos\theta)$
	$v^2 = u^2 - 14 + 14\cos\theta$	A1 **[4]**	Convincing
(b)	Expression in (a) used with $v = 0$ when $\theta = 180^0$ (or $\cos\theta = -1$)	M1	
	least value of $u^2 = 28$	A1 **[2]**	cao
(c)	(i) N2L towards centre	M1	Dimensionally correct equation, T and $mg\cos\theta$ opposing
	$T - mg\cos\theta = \frac{mv^2}{r}$	A1	
	$T = \frac{7m}{5}(32\cdot 2 - 14 + 14\cos\theta) + 9\cdot 8m\cos\theta$	m1	Sub. for v^2 with or without $u^2 = 32\cdot 2$
	$T = \begin{cases} m(25\cdot 48 + 29\cdot 4\cos\theta) \\ mg(2\cdot 6 + 3\cos\theta) \end{cases}$	A1	oe
	(ii) Rod exerts a thrust when $T < 0$ $m(25\cdot 48 + 29\cdot 4\cos\theta) < 0$	M1	Used
	$\cos\theta < -\frac{25\cdot 48}{29\cdot 4}$ $\left(= -\frac{13}{15} = -0\cdot 866\ldots\right)$	A1	
	Thrust exerted in the range $150\cdot 07^0 < \theta < 209\cdot 93^0$	A1 A1 **[8]**	A1 for sight of either $150.073\ldots$ or $209.926\ldots$ Statement (mathematical or otherwise) to the effect that interval is between the given boundaries.
(d)	**No.** Answer to (c) would remain the same as $\cos\theta$ is independent of m.	E1 **[1]**	
	Total for Question 7	**15**	