GCE AS/A LEVEL

2300U10-1

WEDNESDAY, 17 MAY 2023 – MORNING

MATHEMATICS – AS unit 1

PURE MATHEMATICS A

2 hours 30 minutes plus your additional time allowance

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- **a WJEC pink 16-page answer booklet;**
- **a Formula Booklet;**
- **a calculator.**

INSTRUCTIONS TO CANDIDATES

Use black ink, black ball-point pen or your usual method.

Answer ALL questions.

Write your answers in the separate answer booklet provided, following the instructions on the front of the answer booklet.

Use both sides of the paper. Please only write within the white areas of the booklet.

Write the question number in the left-hand margin at the start of each answer. Write the sub parts, e.g. (a), (b) and (c), within the white areas of the booklet.

Leave at least two line spaces between each answer.

(Turn over)

Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

(Turn over)

ADDITIONAL FORMULAE FOR 2023

LAWS OF LOGARITHMS

$$\log_a x + \log_a y \equiv \log_a (xy)$$

$$\log_a x - \log_a y \equiv \log_a \left(\frac{x}{y} \right)$$

$$k \log_a x \equiv \log_a (x^k)$$

SEQUENCES

General term of an arithmetic progression:

$$u_n = a + (n - 1) d$$

General term of a geometric progression:

$$u_n = ar^{n-1}$$

(Turn over)

MENSURATION

For a circle of radius, r, where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A :

$$s = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

(Turn over)

CALCULUS AND DIFFERENTIAL EQUATIONS

DIFFERENTIATION

FUNCTION	DERIVATIVE
$f(x)\,g(x)$	$f'(x)\,g(x) + f(x)\,g'(x)$
$f(g(x))$	$f'(g(x))\,g'(x)$

INTEGRATION

FUNCTION	INTEGRAL
$f'(g(x))\,g'(x)$	$f(g(x)) + c$

Area under a curve = $\displaystyle\int_a^b y\,\mathrm{d}x$

(Turn over)

REMINDER: Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.

01 (a)

Using the binomial theorem, write down and simplify the first three terms in the expansion of $(1 - 3x)^9$ in ascending powers of x **[3 marks]**

(b) Hence, by writing $x = 0 \cdot 001$ in your expansion in part (a), find an approximate value for $(0 \cdot 997)^9$. Show all your working and give your answer correct to three decimal places. **[3 marks]**

02. Solve the following equation for values of θ between $0°$ and $360°$ **[7 marks]**

$$3\sin^2\theta - 5\cos^2\theta = 2\cos\theta$$

(Turn over)

03. The point A has coordinates $(-2, 5)$ and the point B has coordinates $(3, 8)$. The point C lies on the x-axis such that AC is perpendicular to AB

(a) Find the equation of AB [3 marks]

(b) Show that C has coordinates $(1, 0)$ [3 marks]

(c) Calculate the area of triangle ABC [4 marks]

(d) Find the equation of the circle which passes through the points A, B and C [5 marks]

(Turn over)

04 (a)

Find the remainder when the polynomial $3x^3 + 2x^2 + x - 1$ is divided by $(x - 3)$

[3 marks]

(b) **The polynomial $f(x) = 2x^3 - 3x^2 + ax + 6$ is divisible by $(x + 2)$, where a is a real constant.**

(i) **Find the value of a** **[3 marks]**

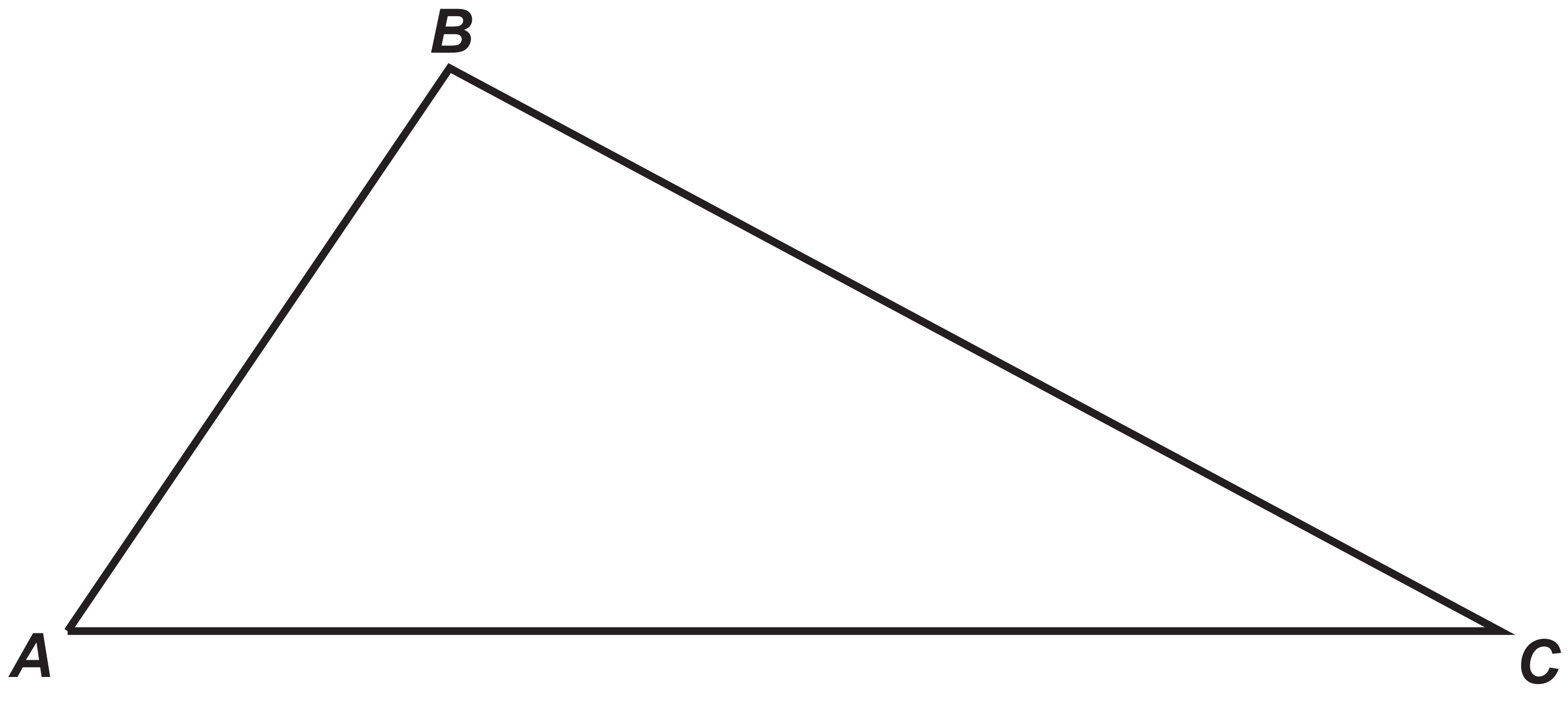
(ii) **Showing all your working, solve the equation $f(x) = 0$** **[4 marks]**

05. **Simplify the expression**

$$\sqrt[3]{512a^2} - \frac{a^{\frac{7}{6}} \times a^{-\frac{1}{3}}}{a^{\frac{1}{6}}}$$

[4 marks]

(Turn over)


B
A
C

06. The diagram opposite shows a triangle ABC

Given that

$$AB = 3,\ BC = 2\sqrt{5},\ AC = 4 + \sqrt{3}$$

find the value of $\cos A\hat{B}C$

Show all your working and give your answer in the form

$$\frac{(a - b\sqrt{3})}{6\sqrt{5}}$$ where a, b are integers.

[7 marks]

07. The curve C has equation $y = 2x^2 + 5x - 12$ and the line L has equation $y = mx - 14$ where m is a real constant.

(a) Given that L is a tangent to C

(i) show that m satisfies the equation

$$m^2 - 10m + 9 = 0$$

[5 marks]

(ii) find the coordinates of the two possible points of contact of C and L [6 marks]

(b) Given instead that L intersects C at two distinct points, find the range of values of m [2 marks]

(Turn over)

08. **Show, by counter example, that the following statement is false.**

"For all positive integer values of n, $n^2 + 1$ is a prime number." **[3 marks]**

09 (a)

Given that $y = x^2 - 3x$ find $\frac{dy}{dx}$ from first principles. **[5 marks]**

(b) **The function f is defined by**

$$f(x) = 4x^{\frac{3}{2}} + \frac{6}{\sqrt{x}} \text{ for } x > 0$$

(i) **Find $f'(x)$** **[2 marks]**

(ii) **When $x > k$, $f(x)$ is an increasing function.**
Determine the least possible value of k
Give your answer correct to two decimal places.
[4 marks]

(Turn over)

10. **Solve the following equations for values of x**

(a) $\ln(2x + 5) = 3$ **[2 marks]**

(b) $5^{2x+1} = 14$ **[3 marks]**

(c) $3\log_7(2x) - \log_7(8x^2) + \log_7 x = \log_3 81$

[6 marks]

11. The function f is defined by $f(x) = \frac{8}{x^2}$

(a) Sketch the graph of $y = f(x)$ [2 marks]

(b) On a separate set of axes, sketch the graph of

$y = f(x - 2)$

Indicate the vertical asymptote and the point where the curve crosses the y-axis. [3 marks]

(c) Sketch the graphs of $y = \frac{8}{x}$ and

$y = \frac{8}{(x-2)^2}$ on the same set of axes.

Hence state the number of roots of the equation

$$\frac{8}{(x-2)^2} = \frac{8}{x}$$ [2 marks]

12. The position vectors of the points A and B relative to a fixed origin O are given by

$$\underline{a} = -3\underline{i} + 4\underline{j} \qquad \underline{b} = 5\underline{i} + 8\underline{j}$$

respectively.

(a) Find the vector $\underline{AB}$ [2 marks]

(b) (i)

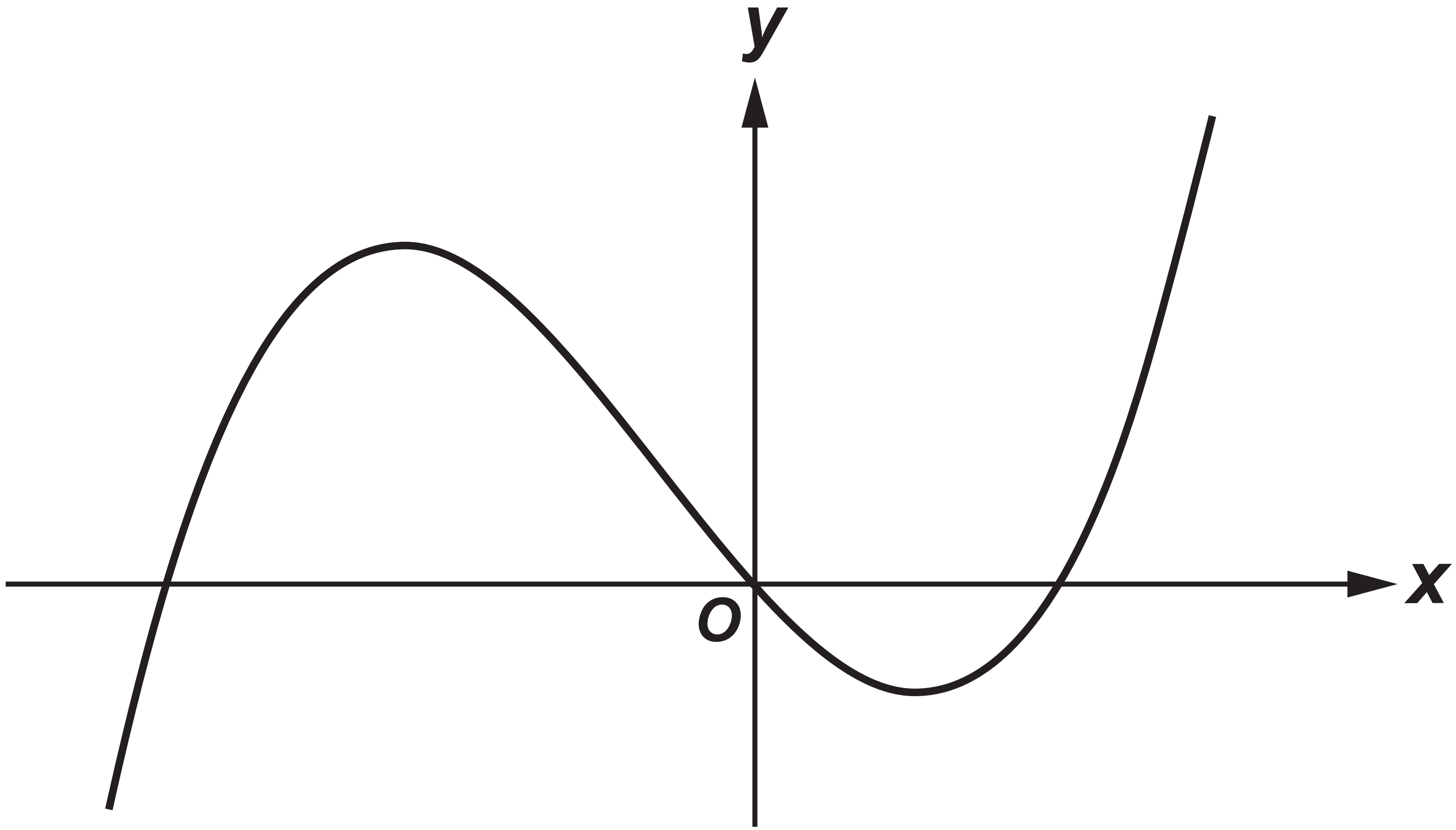
Find a unit vector in the direction of $\underline{a}$ [2 marks]

(ii) The point C is such that the vector $\underline{OC}$ is in the direction of $\underline{a}$

Given that the length of $\underline{OC}$ is 7 units, write down the position vector of C [1 mark]

(c) Calculate the angle AOB [3 marks]

(Turn over)


y
x
O

13 (a)

Find $\int \left(4x^{-\frac{2}{3}} + 5x^{3} + 7 \right) dx$

(b) The diagram opposite shows the graph of

$$y = x(x + 6)(x - 3)$$

Calculate the total area of the regions enclosed by the graph and the x-axis. [9 marks]

(Turn over)

14 (a) Two variables, x and y are such that the rate of change of y with respect to x is proportional to y. State a model which may be appropriate for y in terms of x **[1 mark]**

(b) The concentration, Y units, of a certain drug in a patient's body decreases exponentially with respect to time. At time t hours the concentration can be modelled by $Y = A\mathrm{e}^{-kt}$ where A and k are constants.

A patient was given a dose of the drug that resulted in an initial concentration of 5 units.

(i) After 4 hours, the concentration had dropped to $1{\cdot}25$ units. Show that $k = 0{\cdot}3466$ correct to four decimal places. **[2 marks]**

(ii) The minimum effective concentration of the drug is $0{\cdot}6$ units. How much longer would it take for the drug concentration to drop to the minimum effective level? **[3 marks]**

END OF PAPER