GCE AS/A LEVEL

2300U10-1

WEDNESDAY, 15 MAY 2024 – MORNING

MATHEMATICS – AS unit 1

PURE MATHEMATICS A

2 hours 30 minutes plus your additional time allowance

Surname _____

First name(s) _____

Centre Number _____

Candidate Number 2 _____

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	4	
2	3	
3	3	
4	3	
5	4	
6	7	
7	11	
8	4	
9	9	
10	6	
11	4	
12	10	
13	8	
14	8	
15	7	
16	10	
17	7	
18	12	
Total	120	

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- **a Formula Booklet;**
- **a calculator.**

INSTRUCTIONS TO CANDIDATES

Use black ink, black ball-point pen or your usual method.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces provided on the front cover.

Answer ALL questions.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

(Turn over)

REMINDER: Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.

1. Given that $y = 12\sqrt{x} - \frac{27}{x} + 4$

find the value of $\frac{dy}{dx}$ when $x = 9$

[4 marks]

(Turn over)

2. Find all values of θ in the range $0° < \theta < 180°$ that satisfy the equation $2\sin 2\theta = 1$ [3 marks]

8

(Turn over)

3. Find $\int \left(5x^{\frac{1}{4}} + 3x^{-2} - 2\right) dx$

[3 marks]

10

(Turn over)

4. Given that n is an integer such that

$$1 \leqslant n \leqslant 6$$

use proof by exhaustion to show that $n^2 - 2$ is not divisible by 3

[3 marks]

(Turn over)

5. A triangle ABC has sides $AB = 6\,\text{cm}$, $BC = 11\,\text{cm}$ and $AC = 13\,\text{cm}$

Calculate the area of the triangle.

[4 marks]

(Turn over)

(Turn over)

6 (a) Find the exact value of x that satisfies the equation

$$\frac{7x^{\frac{5}{4}}}{x^{\frac{1}{2}}} = \sqrt{147}$$

[4 marks]

(Turn over)

6 (b) Show that $\frac{(8x - 18)}{(2\sqrt{x} - 3)}$

where $x \neq \frac{9}{4}$

may be written as $2(2\sqrt{x} + 3)$

[3 marks]

(Turn over)

7 (a) The line L_1 passes through the points $A(-3, 0)$ and $B(1, 4)$

Determine the equation of L_1

[3 marks]

(Turn over)

7 (b) The line L_2 has equation

$$y = 3x - 3$$

(i) Given that L_1 and L_2 intersect at the point C find the coordinates of C

(ii) The line L_2 crosses the x-axis at the point D

Show that the coordinates of D are $(1, 0)$ [4 marks]

(Turn over)

(Turn over)

(Turn over)

7 (c) Calculate the area of triangle *ACD*

[2 marks]

(Turn over)

7 (d) Determine the angle *ACD*

[2 marks]

(Turn over)

8. Prove that $x - 10 < x^2 - 5x$ for all real values of x [4 marks]

(Turn over)

(Turn over)

9 (a) Write down the binomial expansion of $(2 - x)^6$ up to and including the term in x^2 [3 marks]

(Turn over)

$$(1 + ax)(2 - x)^6 \equiv 64 + bx + 336x^2 + \ldots$$

9 (b) Given the equation opposite, find the values of the constants a, b

[6 marks]

(Turn over)

(Turn over)

(Turn over)

10. Water is being emptied out of a sink. The depth of water, y cm, at time t seconds, may be modelled by

$$y = t^2 - 14t + 49 \qquad 0 \leqslant t \leqslant 7$$

(a) Find the value of t when the depth of water is 25 cm [3 marks]

(Turn over)

10 (b) Find the rate of decrease of the depth of water when $t = 3$

[3 marks]

(Turn over)

11 (a) Sketch the graph of $y = 3^x$

Clearly label the coordinates of the point where the graph crosses the y-axis. [2 marks]

(Turn over)

11 (b) On the SAME SET OF AXES, sketch the graph of $y = 3^{(x+1)}$ clearly labelling the coordinates of the point where the graph crosses the y-axis.

[2 marks]

12. A curve C has equation

$$y = -x^3 + 12x - 20$$

(a) Find the coordinates of the stationary points of C and determine their nature. [7 marks]

(Turn over)

(Turn over)

12 (b)

Determine the range of values of x for which the curve is decreasing. Give your answer in set notation.

[3 marks]

(Turn over)

13. The position vectors of the points A and B relative to a fixed origin O are given by

$\underline{a} = 4\underline{i} + 7\underline{j}$ $\underline{b} = \underline{i} + 3\underline{j}$

respectively.

(a) Find the vector $\underline{AB}$ [2 marks]

(Turn over)

13 (b) Determine the distance between the points A and B [2 marks]

13 (c)

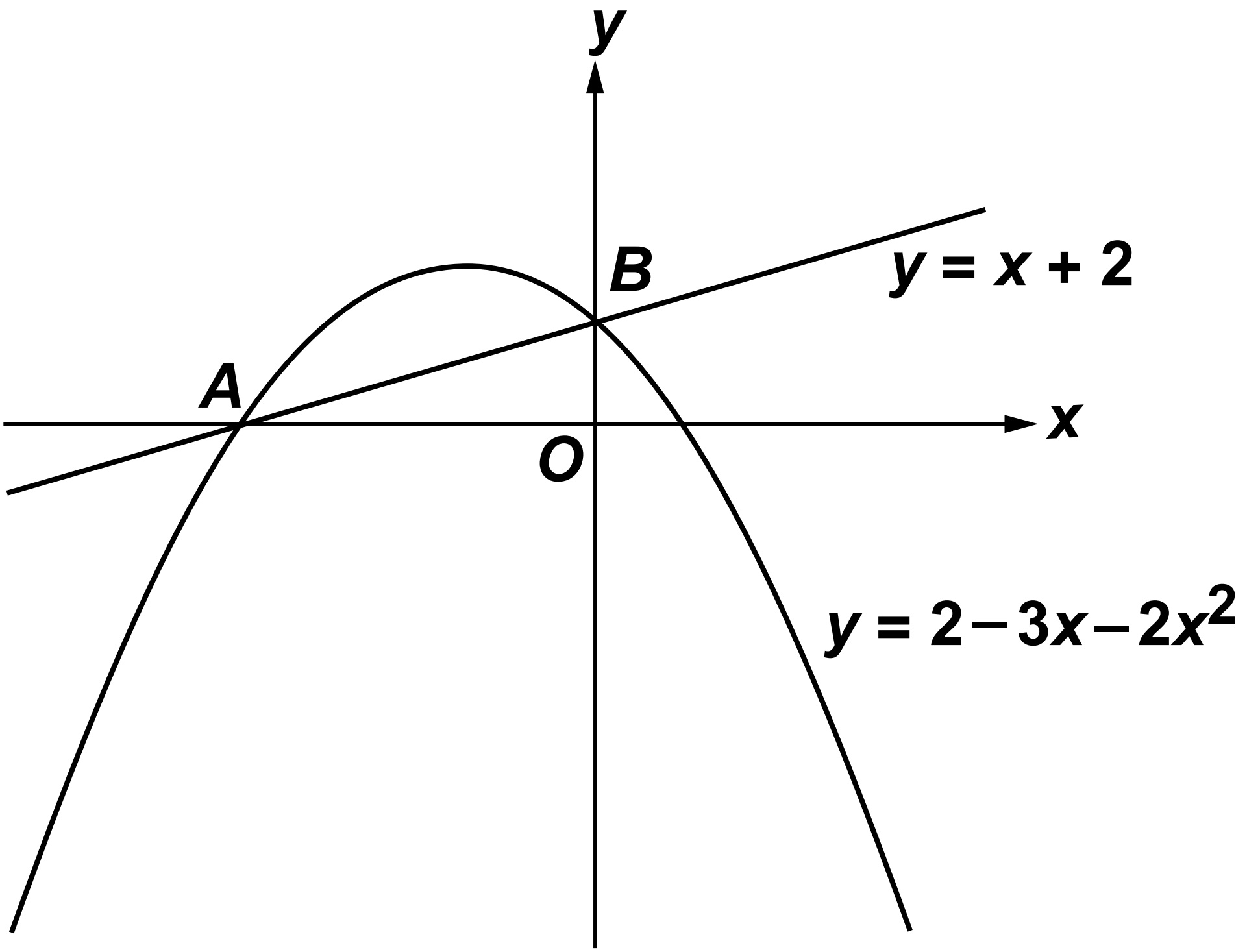
The position vector of the point C is given by $\underline{c} = -2\underline{i} + 5\underline{j}$

The point D is such that the distance between C and D is equal to the distance between A and B and CD is parallel to AB

Find the possible position vectors of the point D [4 marks]

(Turn over)

(Turn over)


y
B
y = x + 2
A
x
O
y = 2 – 3x – 2x²

14. The diagram opposite shows a sketch of the curve C with equation $y = 2 - 3x - 2x^2$ and the line L with equation $y = x + 2$

The curve and the line intersect the coordinate axes at the points A and B

(a) Write down the coordinates of A and B [2 marks]

57

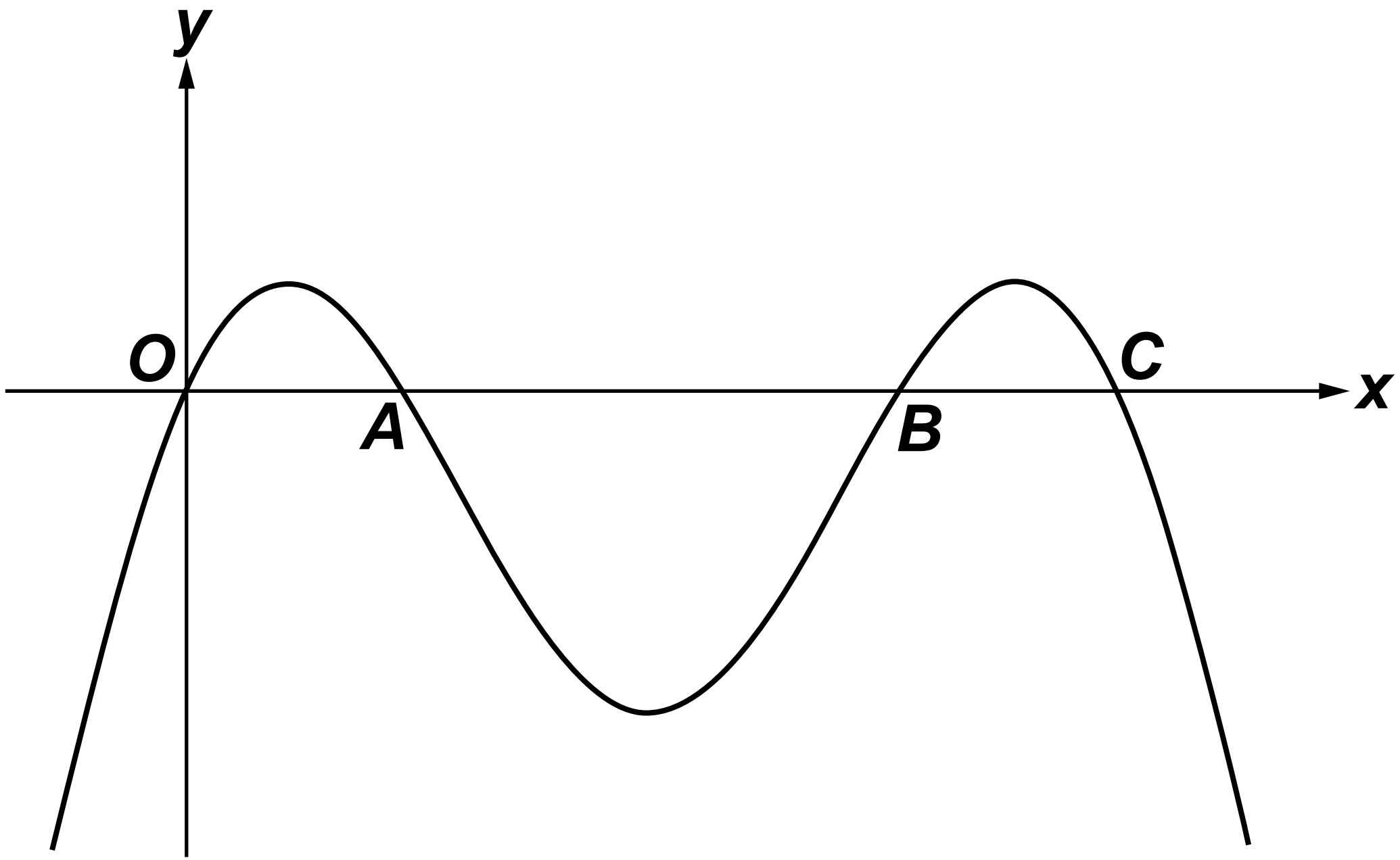
(Turn over)

14 (b) Calculate the area enclosed by C and L [6 marks]

(Turn over)

(Turn over)

(Turn over)


y
O
A
B
C
x

15. The diagram opposite shows a sketch of part of the curve with equation

$$y = 2\sin x + 3\cos^2 x - 3$$

The curve crosses the X-axis at the points O, A, B and C

Find the value of x at each of the points A, B and C [7 marks]

(Turn over)

(Turn over)

(Turn over)

16 (a) Find the range of values of k for which the quadratic equation $x^2 - kx + 4 = 0$ has no real roots. [4 marks]

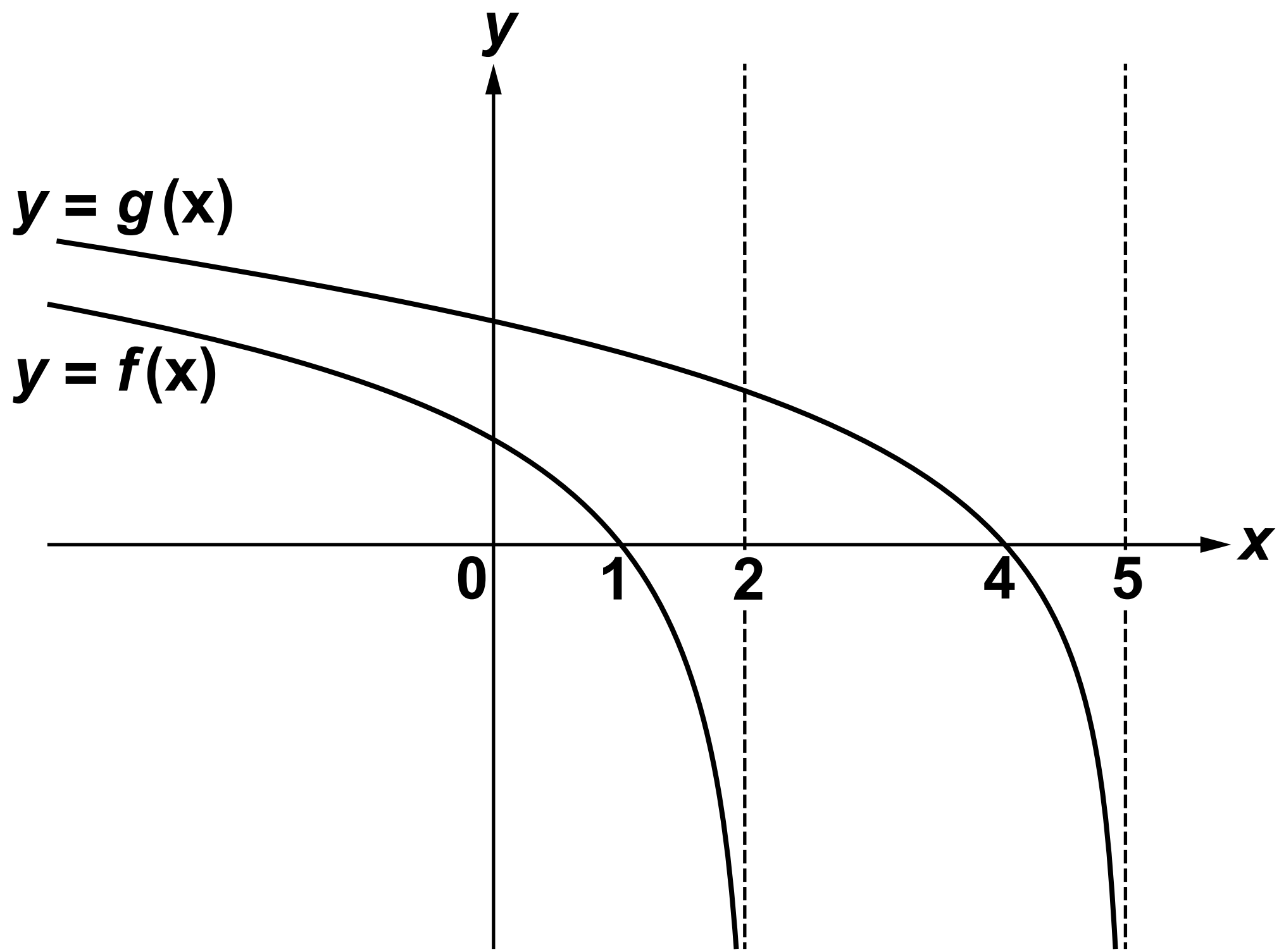
(Turn over)

16 (b) Determine the coordinates of the points of intersection of the graphs of $y = x^2 - 3x + 4$ and $y = x + 16$ [4 marks]

(Turn over)

16 (c) Using the information obtained in parts (a) and (b), sketch the graphs of $y = x^2 - 3x + 4$ and $y = x + 16$ on the same set of axes. [2 marks]

(Turn over)


y
y = g(x)
y = f(x)
x
0
1
2
4
5

17. A function f is defined by

$$f(x) = \log_{10}(2 - x)$$

Another function g is defined by

$$g(x) = \log_{10}(5 - x)$$

The diagram opposite shows a sketch of the graphs of $y = f(x)$ and $y = g(x)$

(a) The point $(c, 1)$ lies on $y = f(x)$

Find the value of c [2 marks]

(Turn over)

17 (b) A point P lies on $y = f(x)$ and has x-coordinate α

Another point Q lies on $y = g(x)$ and also has x-coordinate α

The distance between P and Q is $1 \cdot 2$ units. Find the value of α giving your answer correct to three decimal places. [5 marks]

(Turn over)

(Turn over)

18 (a)

A circle C has centre $(-3, -1)$ and radius $\sqrt{5}$

Show that the equation of C can be written as

$$x^2 + y^2 + 6x + 2y + 5 = 0$$

[2 marks]

(Turn over)

18 (b) (i) Find the equations of the tangents to C that pass through the origin O [6 marks]

(ii) Determine the coordinates of the points where the tangents touch the circle. [4 marks]

(Turn over)

(Turn over)

(Turn over)

(Turn over)

END OF PAPER

Question number	Additional page, if required. Write the question numbers in the left-hand margin.

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