Surname ______________________________

Forename(s) ______________________________

Centre Number ______________________________

Candidate Number ______________________________

Candidate Signature ______________________________

I declare this is my own work.

A-level

FURTHER MATHEMATICS

Paper 1

7367/1

Wednesday 22 May 2024 Afternoon

Time allowed: 2 hours

At the top of the page, write your surname and forename(s), your centre number, your candidate number and add your signature.

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MATERIALS

- **You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.**
- **You should have a graphical or scientific calculator that meets the requirements of the specification.**

INSTRUCTIONS

- **Use black ink or black ball-point pen. Pencil should only be used for drawing.**
- **Answer ALL questions.**
- **You must answer each question in the space provided for that question.**
- **Do NOT write on blank pages.**
- **If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).**
- **Show all necessary working; otherwise marks for method may be lost.**
- **Do all rough work in this book. Cross through any work that you do not want to be marked.**

INFORMATION

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

ADVICE

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

DO NOT TURN OVER UNTIL TOLD TO DO SO

Answer ALL questions in the spaces provided.

1 The roots of the equation
$20x^3 - 16x^2 - 4x + 7 = 0$ are α, β and γ

Find the value of $\alpha\beta + \beta\gamma + \gamma\alpha$

Circle your answer. [1 mark]

$-\frac{4}{5}$ $-\frac{1}{5}$ $\frac{1}{5}$ $\frac{4}{5}$

2 The complex number $z = \mathrm{e}^{\frac{\mathrm{i}\pi}{3}}$

Which ONE of the following is a real number?

Circle your answer. [1 mark]

z^4 z^5 z^6 z^7

3 The function f is defined by

$\mathrm{f}(x) = x^2 \qquad (x \in \mathbb{R})$

Find the mean value of $\mathrm{f}(x)$ between $x = 0$ and $x = 2$

Circle your answer. [1 mark]

$\frac{2}{3}$ $\quad$ $\frac{4}{3}$ $\quad$ $\frac{8}{3}$ $\quad$ $\frac{16}{3}$

4 Which ONE of the following statements is correct?

Tick (✓) ONE box. [1 mark]

☐ $\lim_{x \to 0} (x^2 \ln x) = 0$

☐ $\lim_{x \to 0} (x^2 \ln x) = 1$

☐ $\lim_{x \to 0} (x^2 \ln x) = 2$

☐ $\lim_{x \to 0} (x^2 \ln x)$ is not defined.

5 **The points A, B and C have coordinates $A(5, 3, 4)$, $B(8, -1, 9)$ and $C(12, 5, 10)$**

The points A, B and C lie in the plane Π

5 (a) **Find a vector that is normal to the plane Π**
[3 marks]

5 (b) **Find a Cartesian equation of the plane Π**
[2 marks]

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6 The sequence $u_1, u_2, u_3, \ldots$ is defined by

$u_1 = 1$

$u_{n+1} = u_n + 3n$

Prove by induction that for all integers $n \geq 1$

$$u_n = \frac{3}{2}n^2 - \frac{3}{2}n + 1$$

[4 marks]

[Turn over]

7 The complex numbers z and w satisfy the simultaneous equations

$z + w^* = 5$

$3z^* - w = 6 + 4\mathrm{i}$

Find z and w [5 marks]

8 **The ellipse *E* has equation**

$$x^2 + \frac{y^2}{9} = 1$$

The line with equation $y = mx + 4$ is a tangent to *E*

Without using differentiation show that $m = \pm\sqrt{7}$ [4 marks]

9 (a) **It is given that**

$$p = \ln\left(r + \sqrt{r^2 + 1}\right)$$

Starting from the exponential definition of the sinh function, show that $\sinh p = r$ [4 marks]

9 (b) **Solve the equation**

$\cosh^2 x = 2\sinh x + 16$

Give your answers in LOGARITHMIC FORM.
[4 marks]

10 The complex numbers z and w are defined by

$$z = \cos\frac{\pi}{4} + \mathrm{i}\sin\frac{\pi}{4}$$

and $$w = \cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}$$

By evaluating the product zw, show that

$\tan\frac{5\pi}{12} = 2 + \sqrt{3}$ [6 marks]

11 (a) Find $\frac{\mathrm{d}}{\mathrm{d}x}(x^2\tan^{-1}x)$ [1 mark]

11 (b) Hence find $\int 2x\tan^{-1}x\,\mathrm{d}x$ [4 marks]

12 The line L_1 has equation

$$\mathbf{r} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$$

The transformation T is represented by the matrix

$$\begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 6 \\ -5 & 2 & -3 \end{bmatrix}$$

The transformation T transforms the line L_1 to the line L_2

12 (a) Show that the angle between L_1 and L_2 is 0.701 radians, correct to three decimal places. [4 marks]

12 (b) **Find the shortest distance between L_1 and L_2**

Give your answer in an exact form. [6 marks]

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13 (a) **Use de Moivre's theorem to show that**

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ [3 marks]

13 (b) Use de Moivre’s theorem to express $\sin 3\theta$ in terms of $\sin \theta$ [2 marks]

13 (c) **Hence show that**

$$\cot 3\theta = \frac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta - 1}$$ **[4 marks]**

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14 **Solve the differential equation**

$$\frac{dy}{dx} + y\tanh x = \sinh^3 x$$

given that $y = 3$ when $x = \ln 2$

Give your answer in an exact form. [7 marks]

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[Turn over]

15 **A curve is defined parametrically by the equations**

$$x = \frac{3}{2}t^3 + 5$$

$$y = t^{\frac{9}{2}} \qquad (t \geq 0)$$

Show that the arc length of the curve from $t = 0$ to $t = 2$ is equal to 26 units. [5 marks]

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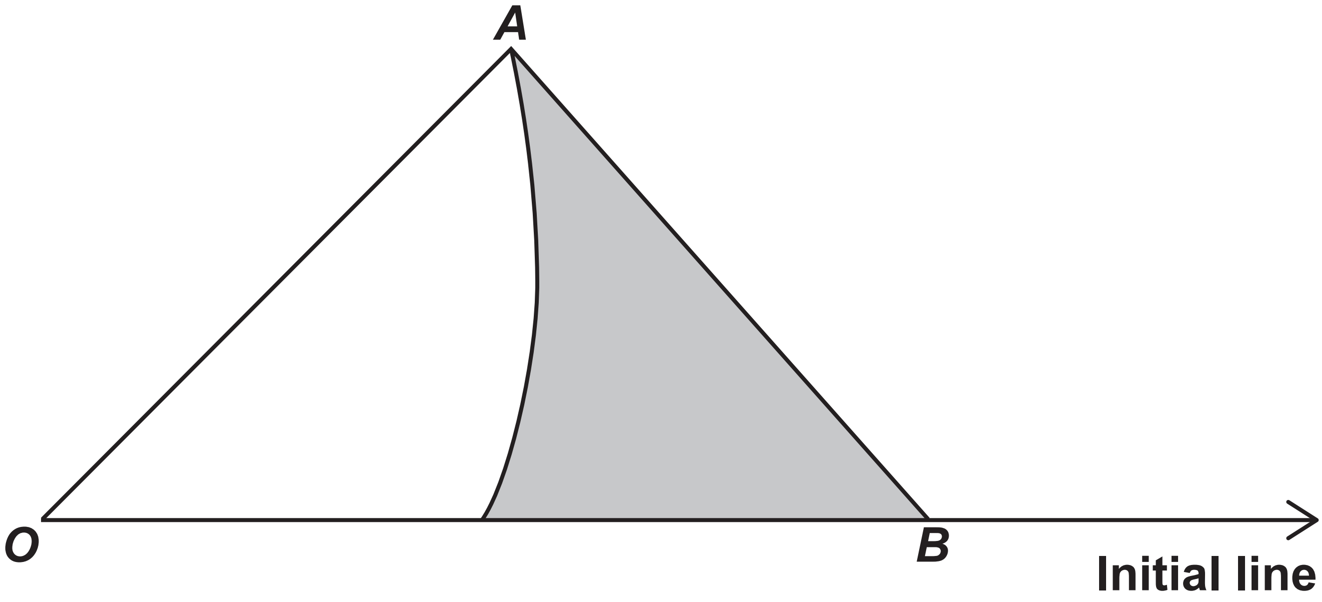
[Turn over]

16 **The curve *C* has polar equation $r = 2 + \tan\theta$**

The curve *C* meets the line $\theta = \frac{\pi}{4}$ at the point *A*

The point *B* has polar coordinates (4, 0)

The diagram shows part of the curve *C*, and the points *A* and *B*



16 (a) **Show that the area of triangle *OAB* is $3\sqrt{2}$ units. [2 marks]**

16 (b) **Find the area of the shaded region.**

Give your answer in an exact form. [7 marks]

17 **By making a suitable substitution, show that**

$$\int_{-2}^{1} \sqrt{x^2 + 6x + 8}\ \mathrm{d}x = 2\sqrt{15} - \frac{1}{2}\cosh^{-1}(4)$$

[7 marks]

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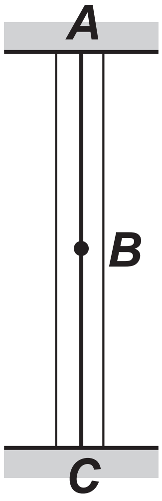
[Turn over]

18 **IN THIS QUESTION USE $g = 9.8 \text{ m s}^{-2}$**

Two light elastic strings each have one end attached to a small ball *B* of mass 0.5 kg

The other ends of the strings are attached to the fixed points *A* and *C*, which are 8 metres apart with *A* vertically above *C*

The whole system is in a thin tube of oil, as shown in the diagram below.



The string connecting *A* and *B* has natural length 2 metres, and the tension in this string is $7e$ newtons when the extension is e metres.

The string connecting *B* and *C* has natural length 3 metres, and the tension in this string is $3e$ newtons when the extension is e metres.

18 (a) **Find the extension of each string when the system is in equilibrium. [3 marks]**

18 (b) **It is known that in a large bath of oil, the oil causes a resistive force of magnitude $4.5v$ newtons to act on the ball, where v m s^{-1} is the speed of the ball.**

Use this model to answer part (b)(i) and part (b)(ii).

18 (b) (i) The ball is pulled a distance of 0.6 metres downwards from its equilibrium position towards *C*, and released from rest.

Show that during the subsequent motion the particle satisfies the differential equation

$$\frac{d^2x}{dt^2} + 9\frac{dx}{dt} + 20x = 0$$

where x metres is the displacement of the particle below the equilibrium position at time t seconds after the particle is released.
[3 marks]

18 (b) (ii) **Find x in terms of t [5 marks]**

18 (c) **State one limitation of the model used in part (b) [1 mark]**

END OF QUESTIONS

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