AQA

Surname _____

Forename(s) _____

Centre Number _____

Candidate Number _____

Candidate Signature _____

I declare this is my own work.

A-level

MATHEMATICS

Paper 1

7357/1

Tuesday 4 June 2024 Afternoon

Time allowed: 2 hours

At the top of the page, write your surname and forename(s), your centre number, your candidate number and add your signature.

[Turn over]

JUN247357101

MATERIALS

- **You must have the AQA Formulae for A-level Mathematics booklet.**
- **You should have a graphical or scientific calculator that meets the requirements of the specification.**

INSTRUCTIONS

- **Use black ink or black ball-point pen. Pencil should only be used for drawing.**
- **Answer ALL questions.**
- **You must answer each question in the space provided for that question.**
- **If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).**
- **Do NOT write on blank pages.**
- **Show all necessary working; otherwise marks for method may be lost.**
- **Do all rough work in this book. Cross through any work that you do not want to be marked.**

INFORMATION

- **The marks for questions are shown in brackets.**
- **The maximum mark for this paper is 100.**

ADVICE

- **Unless stated otherwise, you may quote formulae, without proof, from the booklet.**
- **You do not necessarily need to use all the space provided.**

DO NOT TURN OVER UNTIL TOLD TO DO SO

Answer ALL questions in the spaces provided.

1 **Find the coefficient of x in the expansion of**

$$(4x^3 - 5x^2 + 3x - 2)(x^5 + 4x + 1)$$

Circle your answer. [1 mark]

–5 **–2** **7** **11**

2 The function f is defined by $f(x) = e^x + 1$ for $x \in \mathbb{R}$

Find an expression for $f^{-1}(x)$

Tick (✓) ONE box. [1 mark]

☐ $f^{-1}(x) = \ln(x - 1)$

☐ $f^{-1}(x) = \ln(x) - 1$

☐ $f^{-1}(x) = \dfrac{1}{e^x + 1}$

☐ $f^{-1}(x) = \dfrac{x - 1}{e}$

3 The expression

$$\frac{12x^2 + 3x + 7}{3x - 5}$$

can be written as

$$Ax + B + \frac{C}{3x - 5}$$

State the value of A

Circle your answer. [1 mark]

3 4 7 9

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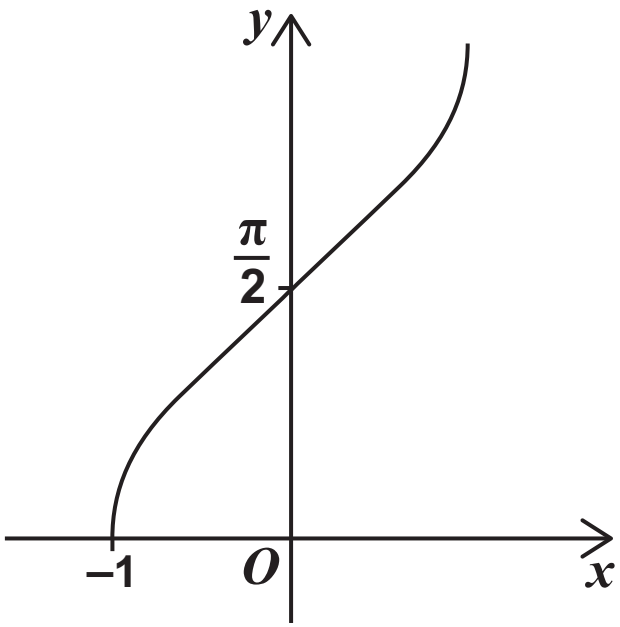
[Turn over]

4 **One of the diagrams below and on the opposite page, shows the graph of $y = \arccos x$**

Identify the graph of $y = \arccos x$

Tick (✓) ONE box. [1 mark]

☐



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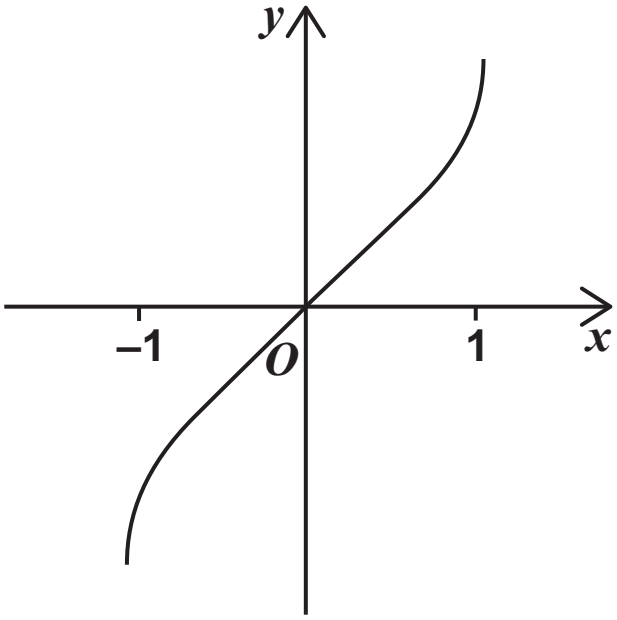

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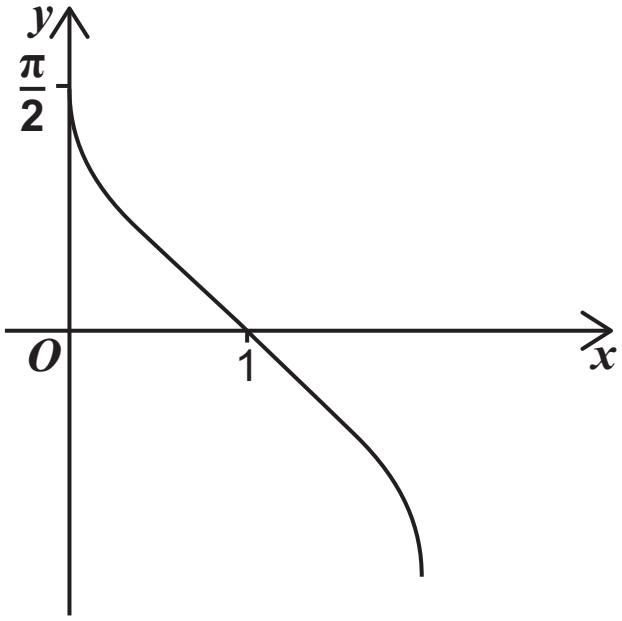

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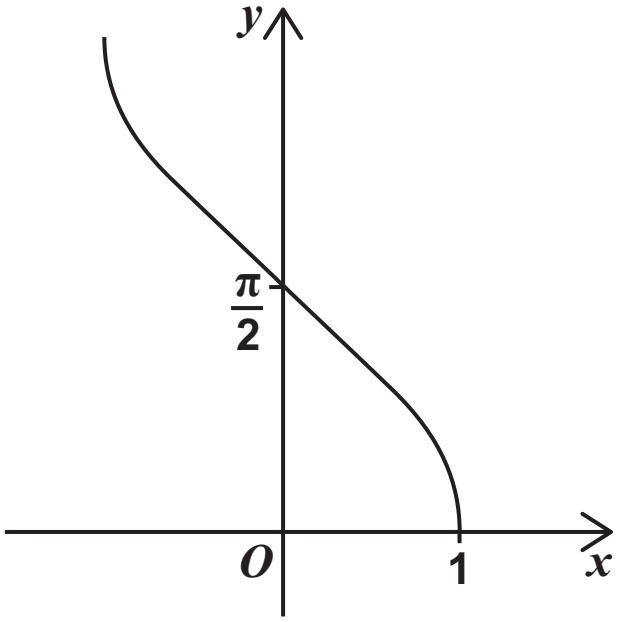

[Turn over]

5 **Solve the equation**

$\sin^2 x = 1$

for $0° < x < 360°$ [3 marks]

[Turn over]

6 Use the chain rule to find $\frac{dy}{dx}$ when $y = (x^3 + 5x)^7$ [2 marks]

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[Turn over]

7 **Show that**

$$\frac{3 + \sqrt{8n}}{1 + \sqrt{2n}}$$

can be written as

$$\frac{4n - 3 + \sqrt{2n}}{2n - 1}$$

where n is a positive integer. [4 marks]

8 (a) Find the first three terms, in ascending powers of x, in the expansion of

$(2 + kx)^5$

where k is a positive constant. [3 marks]

8 (b) **Hence, given that the coefficient of x is four times the coefficient of x^2, find the value of k [2 marks]**

9 (a) **Show that, for small values of θ measured in radians**

$$\cos 4\theta + 2\sin 3\theta - \tan 2\theta \approx A + B\theta + C\theta^2$$

where A, B and C are constants to be found. [3 marks]

[Turn over]

9 (b) Use your answer to part (a) to find an approximation for

$\cos 0.28 + 2 \sin 0.21 - \tan 0.14$

Give your answer to three decimal places.
[2 marks]

[Turn over]

10 (a) **An arithmetic sequence has 300 terms.**

The first term of the sequence is –7 and the last term is 32

Find the sum of the 300 terms. [2 marks]

[Turn over]

10 (b) **A school holds a raffle at its summer fair.**

There are nine prizes.

The total value of the prizes is £1260

The values of the prizes form an arithmetic sequence.

The top prize has the highest value, and the bottom prize has the least value.

The value of the top prize is six times the value of the bottom prize.

Find the value of the top prize. [4 marks]

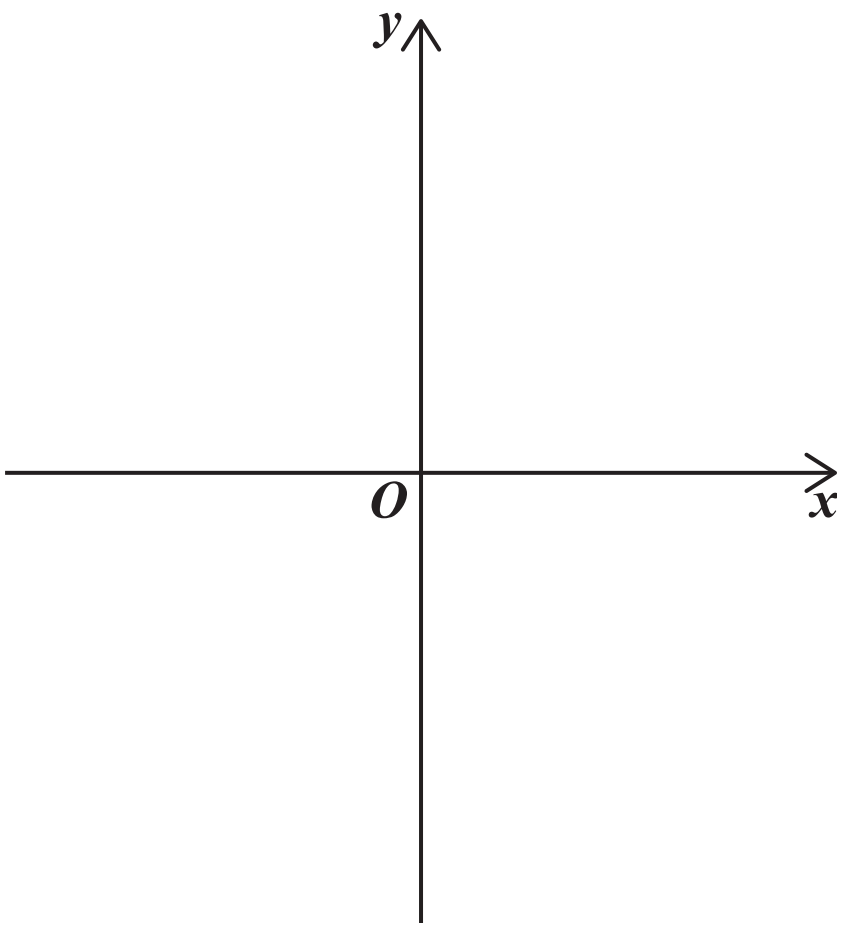
[Turn over]

11 It is given that

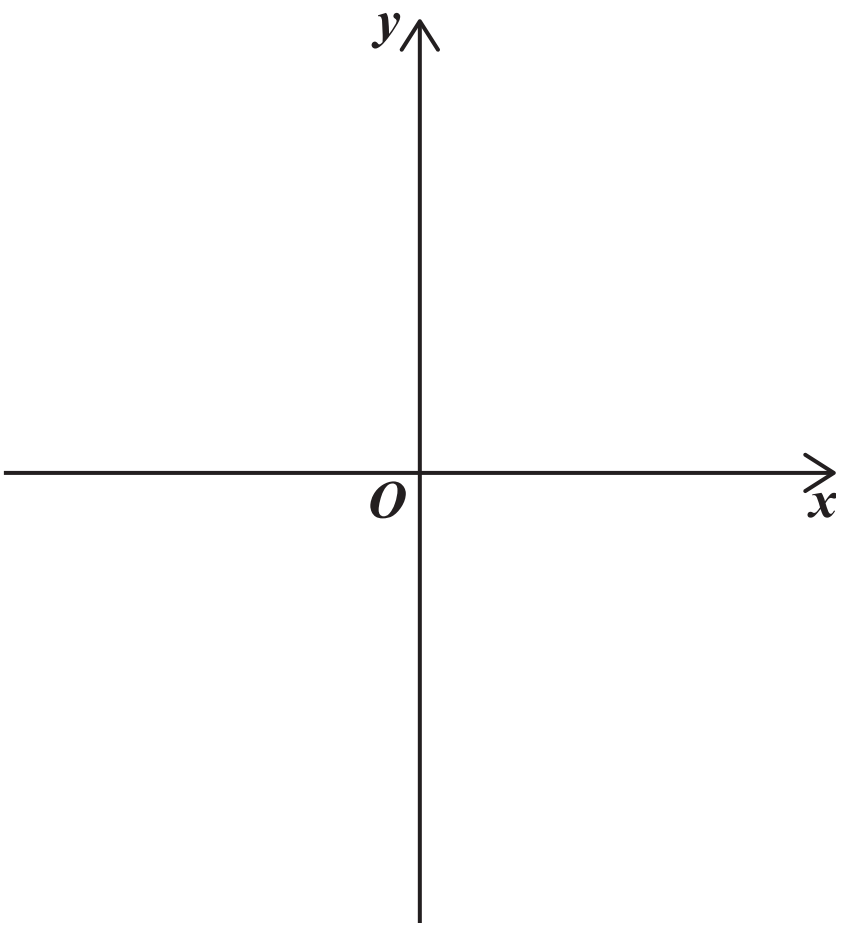
$$f(x) = x(x - a)(x - 6)$$

where $0 < a < 6$

11 (a) Sketch the graph of $y = f(x)$ on the axes below. [3 marks]



11 (b) **Sketch the graph of $y = f(-2x)$ on the axes below. [2 marks]**



[Turn over]

12 The terms, u_n, of a periodic sequence are defined by

$$u_1 = 3 \quad \text{and} \quad u_{n+1} = \frac{-6}{u_n}$$

12 (a) Find u_2, u_3 and u_4 [2 marks]

12 (b) State the period of the sequence. [1 mark]

[Turn over]

12 (c) Find the value of $\sum_{n=1}^{101} u_n$ [2 marks]

13 (a) It is given that

$$P(x) = 4x^3 + 8x^2 + 11x + 4$$

Use the factor theorem to show that $(2x + 1)$ is a factor of $P(x)$ [2 marks]

13 (b) Express $\mathrm{P}(x)$ in the form

$$\mathrm{P}(x) = (2x + 1)(ax^2 + bx + c)$$

where a, b and c are constants to be found.
[2 marks]

13 (c) Given that n is a positive integer, use your answer to part (b) to explain why $4n^3 + 8n^2 + 11n + 4$ is NEVER prime.
[2 marks]

14 (a) The equation

$$x^3 = e^{6-2x}$$

has a single solution, $x = \alpha$

By considering a suitable change of sign, show that α lies between 0 and 4 [2 marks]

[Turn over]

14 (b) Show that the equation $x^3 = e^{6-2x}$ can be rearranged to give

$$x = 3 - \frac{3}{2}\ln x$$ [3 marks]

[Turn over]

14 (c) (i) Use the iterative formula

$$x_{n+1} = 3 - \frac{3}{2} \ln x_n$$

with $x_1 = 4$, to find x_2, x_3 and x_4

Give your answers to three decimal places.
[2 marks]

[Turn over]

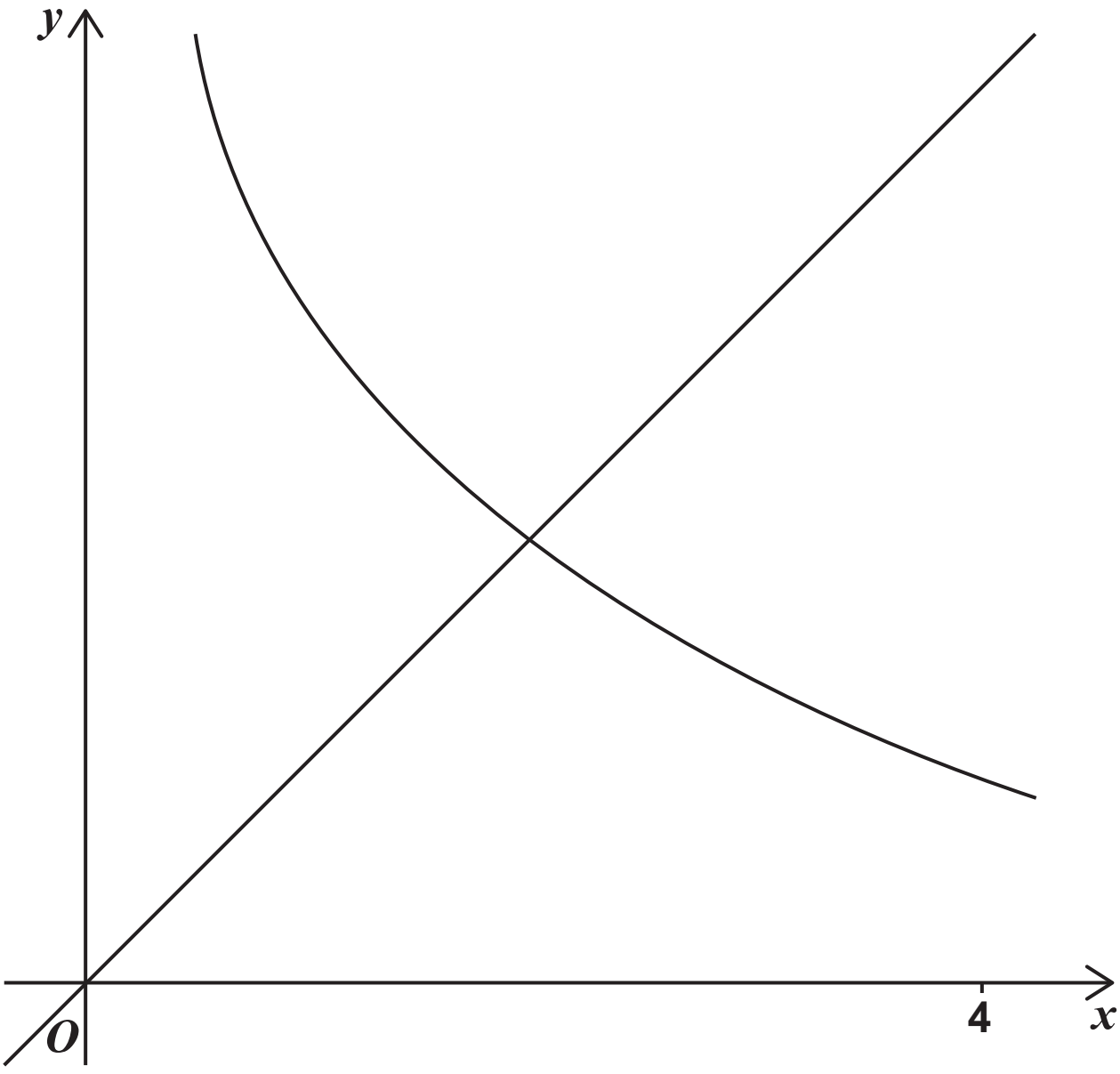
14 (c) (ii) FIGURE 1 below shows a sketch of parts of the graphs of

$y = 3 - \frac{3}{2} \ln x$ and $y = x$

On FIGURE 1, draw a staircase or cobweb diagram to show how convergence takes place.

Label, on the x-axis, the positions of x_2, x_3 and x_4 [2 marks]

FIGURE 1



14 (c) (iii) Explain why the iterative formula

$$x_{n+1} = 3 - \frac{3}{2} \ln x_n$$

fails to converge to α when the starting value is $x_1 = 0$ [1 mark]

15 (a) **Show that the expression**

$\sin 2\theta \operatorname{cosec} \theta + \cos 2\theta \sec \theta$

can be written as

$4 \cos \theta - \sec \theta$

where $\sin \theta \neq 0$ **and** $\cos \theta \neq 0$ **[4 marks]**

[Turn over]

15 (b) **A student is attempting to solve the equation**

$\sin 2\theta \operatorname{cosec} \theta + \cos 2\theta \sec \theta = 3$

for $0° \leq \theta \leq 360°$

They use the result from part (a), and write the following INCORRECT solution:

$\sin 2\theta \operatorname{cosec} \theta + \cos 2\theta \sec \theta = 3$

Step 1 $4 \cos \theta - \sec \theta = 3$

Step 2 $4 \cos \theta - \dfrac{1}{\cos \theta} - 3 = 0$

Step 3 $4 \cos^2 \theta - 3 \cos \theta - 1 = 0$

Step 4 $\cos \theta = 1$ **or** $\cos \theta = -0.25$

Step 5 $\theta = 0°, 104.5°, 255.5°, 360°$

15 (b) (i) **Explain why the student should reject one of their values for cos θ in Step 4. [1 mark]**

15 (b) (ii) **State the correct solutions to the equation**

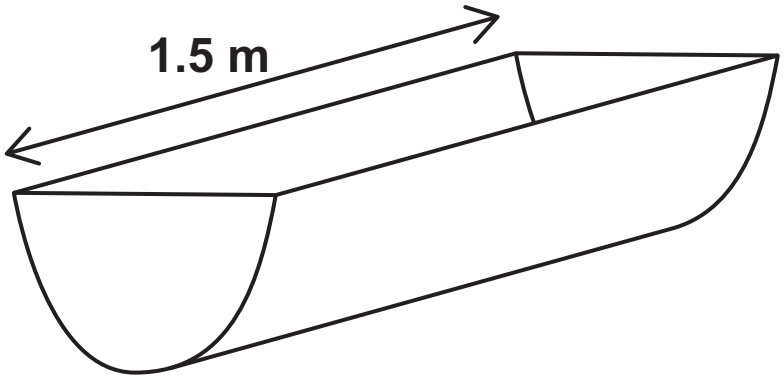
$$\sin 2\theta \operatorname{cosec} \theta + \cos 2\theta \sec \theta = 3$$

for $0° \leq \theta \leq 360°$ [1 mark]

[Turn over]

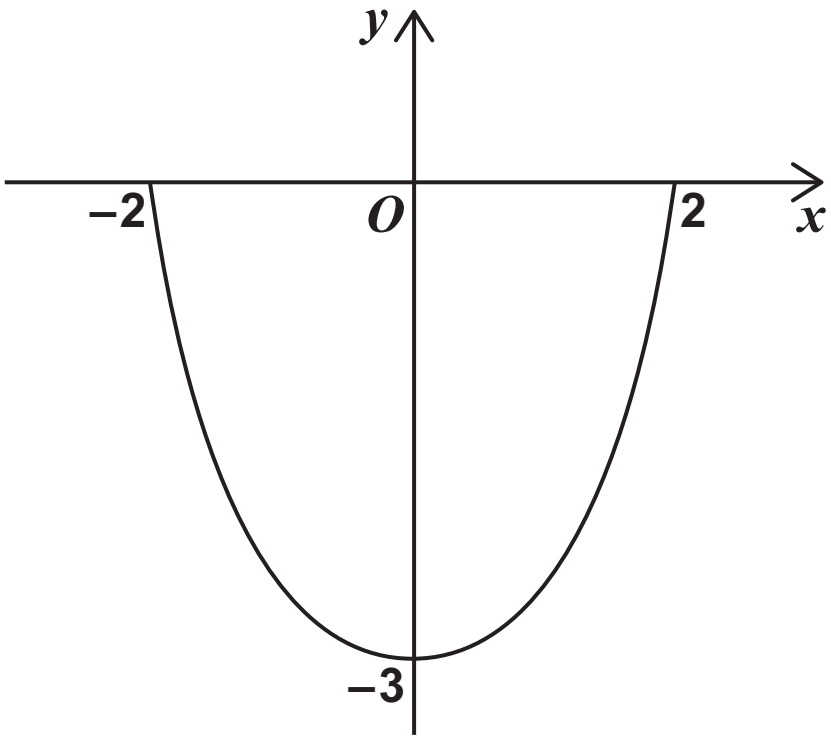
16 **FIGURE 2 below shows a 1.5 metre length of pipe.**

FIGURE 2



The symmetrical cross-section of the pipe is shown below, in FIGURE 3, where x and y are measured in centimetres.

FIGURE 3



Use the trapezium rule, with the values shown in the table below, to find the best estimate for the VOLUME of the pipe.

x	0	0.4	0.8	1.2	1.6	2
y	–3	–2.943	–2.752	–2.353	–1.572	0

[5 marks]

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[Turn over]

17 The function f is defined by

$\mathrm{f}(x) = |x| + 1$ for $x \in \mathbb{R}$

The function g is defined by

$\mathrm{g}(x) = \ln x$

where g has its greatest possible domain.

17 (a) Using set notation, state the range of f
[2 marks]

17 (b) State the domain of $\mathbf{g}$ [1 mark]

17 (c) The composite function $\mathbf{h}$ is given by

$\mathbf{h}(x) = \mathbf{gf}(x)$ for $x \in \mathbb{R}$

17 (c) (i) Write down an expression for $\mathbf{h}(x)$ in terms of x [1 mark]

17 (c) (ii) **Determine if $\mathbf{h}$ has an inverse.**

Fully justify your answer. [2 marks]

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18 (a) **Use a suitable substitution to show that**

$$\int_0^4 (4x+1)(2x+1)^{\frac{1}{2}}\,\mathrm{d}x$$

can be written as

$$\frac{1}{2}\int_a^9 \left(2u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u$$

where a is a constant to be found. [5 marks]

18 (b) **Hence, or otherwise, show that**

$$\int_0^4 (4x+1)(2x+1)^{\frac{1}{2}}\,\mathrm{d}x = \frac{1322}{15}$$ **[4 marks]**

[Turn over]

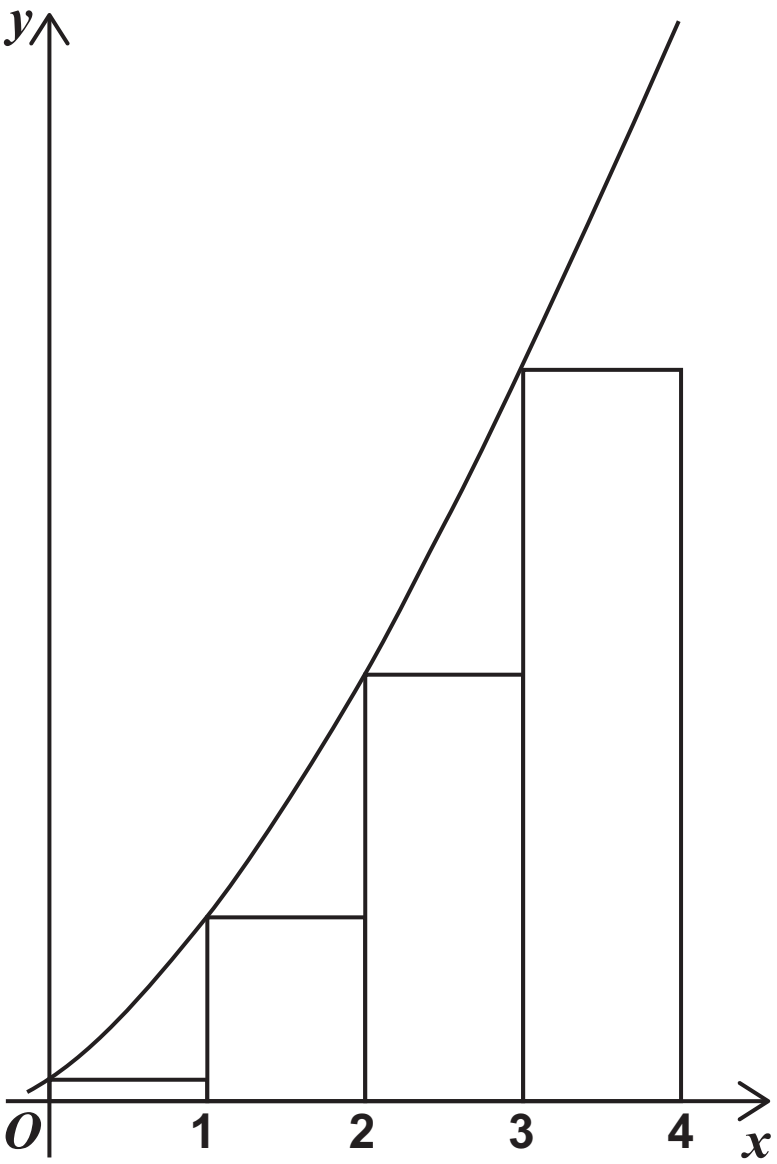
18 (c) **A graph has the equation**

$$y = (4x + 1)\sqrt{2x + 1}$$

A student uses four rectangles to approximate the area under the graph between the lines $x = 0$ and $x = 4$

The rectangles are all the same width.

All the rectangles are drawn under the curve as shown in the diagram below.



The total area of the four rectangles is A

The student decides to improve their approximation by increasing the number of rectangles used.

Explain why the value of the student's improved approximation will be

greater than A, but less than $\frac{1322}{15}$ [2 marks]

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[Turn over]

19 A curve has equation

$$y^3 e^{2x} + 2y - 16x = k$$

where k is a constant.

The curve has a stationary point on the y-axis.

Determine the value of k [7 marks]

20 A gardener stores rainwater in a cylindrical container.

The container has a height of 130 centimetres.

The gardener empties the water from the container through a hose.

The hose is attached 5 centimetres from the bottom of the container.

At time t minutes after the hose is switched on, the depth of water, h centimetres, in the container decreases at a rate which is proportional to $h - 5$

Initially the container of water is full, and the depth of water is decreasing at a rate of 1.5 centimetres per minute.

20 (a) Show that

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -0.012(h - 5)$$ [3 marks]

20 (b) **Solve the differential equation**

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -0.012(h - 5)$$

to find an expression for h in terms of t
[5 marks]

[Turn over]

20 (c) **Find the time taken for the container to be half empty.**

Give your answer to the nearest minute.
[2 marks]

END OF QUESTIONS

Additional page, if required.
Write the question numbers in the left-hand margin.

Additional page, if required.
Write the question numbers in the left-hand margin.

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For Examiner's Use	
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