



**GCE A LEVEL**

**1300U30-1**

**THURSDAY, 8 JUNE 2023 – MORNING**

**MATHEMATICS – A2 unit 3**

**PURE MATHEMATICS B**

**2 hours 30 minutes plus your additional  
time allowance**

## **ADDITIONAL MATERIALS**

**In addition to this examination paper, you will need:**

- **a WJEC pink 16-page answer booklet;**
- **a Formula Booklet;**
- **a calculator.**

## **INSTRUCTIONS TO CANDIDATES**

**Use black ink, black ball-point pen or your usual method.**

**Answer ALL questions.**

**Write your answers in the separate answer booklet provided, following the instructions on the front of the answer booklet.**

**(Turn over)**

**Use both sides of the paper. Please only write within the white areas of the booklet.**

**Write the question number in the left-hand margin at the start of each answer. Write the sub parts, e.g. (a), (b) and (c), within the white areas of the booklet.**

**Leave at least two line spaces between each answer.**

**Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.**

**Answers without working may not gain full credit.**

**Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.**

**(Turn over)**

**INFORMATION FOR CANDIDATES**

**The maximum mark for this paper is 120.**

**The number of marks is given in brackets at the end of each question or part-question.**

**You are reminded of the necessity for good English and orderly presentation in your answers.**

**(Turn over)**

**ADDITIONAL FORMULAE FOR 2023****LAWS OF LOGARITHMS**

$$\log_a x + \log_a y \equiv \log_a (xy)$$

$$\log_a x - \log_a y \equiv \log_a \left( \frac{x}{y} \right)$$

$$k \log_a x \equiv \log_a (x^k)$$

**(Turn over)**

**SEQUENCES**

**General term of an arithmetic progression:**

$$u_n = a + (n - 1) d$$

**General term of a geometric progression:**

$$u_n = ar^{n-1}$$

**(Turn over)**

**MENSURATION**

For a circle of radius,  $r$ , where an angle at the centre of  $\theta$  radians subtends an arc of length  $S$  and encloses an associated sector of area  $A$  :

$$s = r\theta$$

$$A = \frac{1}{2} r^2 \theta$$

# CALCULUS AND DIFFERENTIAL EQUATIONS

## DIFFERENTIATION

**FUNCTION**

**DERIVATIVE**

**$f(x)g(x)$**

**$f'(x)g(x) + f(x)g'(x)$**

**$f(g(x))$**

**$f'(g(x))g'(x)$**

## INTEGRATION

**FUNCTION**

**INTEGRAL**

**$f'(g(x))g'(x)$**

**$f(g(x)) + c$**

Area under a curve =  $\int_a^b y dx$

**(Turn over)**

**REMINDER: Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.**

- 01. The 12th term of an arithmetic series is 41 and the sum of the first 16 terms is 488  
Find the first term and the common difference of the series. [5 marks]**

**(Turn over)**

02 (a)

Differentiate each of the following  
with respect to  $X$

(i)  $(\sin x + x^2)^5$  [2 marks]

(ii)  $x^3 \cos x$  [2 marks]

(iii)  $\frac{e^{3x}}{\sin 2x}$  [3 marks]

(Turn over)

**02 (b)**

**Find the equation of the tangent to the curve**

$$4y^2 - 7xy + x^2 = 12$$

**at the point (2, 4)**

**[6 marks]**

**(Turn over)**

12

03 (a)

Express  $\frac{9}{(1-x)(1+2x)^2}$

in terms of partial fractions.

[4 marks]

(b) USING YOUR ANSWER FROM PART (a), find the expansion of

$\frac{9}{(1-x)(1+2x)^2}$  in ascending

powers of  $x$  as far as the term in  $x^2$   
State the values of  $x$  for which the expansion is valid.

[7 marks]

(Turn over)

04. A function  $f$  with domain  $(-\infty, \infty)$  is defined by

$$f(x) = 6x^3 + 35x^2 - 7x - 6$$

(a) Determine the number of roots of the equation  $f(x) = 0$  in the interval  $[-1, 1]$

[2 marks]

(Turn over)

**04 (b)**

**Use the Newton-Raphson method to find a root of the equation  $f(x) = 0$  Starting with  $x_0 = 1$**

**(i) write down the value of  $x_1$**

**(ii) determine the value of the root correct to one decimal place.**

**[4 marks]**

**(Turn over)**

**04 (c)**

**It is suggested that another iterative sequence**

$$x_{n+1} = \sqrt{\frac{7x_n + 6 - 6x_n^3}{35}}$$

**starting with  $x_0 = -3$ , could be used**

**to find a root of the equation**  
 **$f(x) = 0$**

**Explain why this method fails.**

**[2 marks]**

**(Turn over)**

**05. A tree is 80 cm in height when it is planted. In the first year, the tree grows in height by 32 cm. In each subsequent year, the tree grows in height by 90% of the growth of the previous year.**

**(a) Find the height of the tree 10 years after it was planted. [4 marks]**

**(b) Determine the maximum height of the tree. [2 marks]**

06 (a)

Using the trigonometric identity

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

show that the EXACT value of

$$\cos 75^\circ \text{ is } \frac{\sqrt{6} - \sqrt{2}}{4}$$

[3 marks]

(b) Solve the equation

$$2\cot^2 x + \operatorname{cosec} x = 4$$

for values of  $x$  between  $0^\circ$  and  $360^\circ$

[6 marks]

(Turn over)

06 (c) (i)

Express  $7 \cos \theta - 24 \sin \theta$  in the

form  $R \cos (\theta + \alpha)$ , where  $R$  and  $\alpha$

are constants with  $R > 0$  and

$$0^\circ < \alpha < 90^\circ$$

(ii) Find all values of  $\theta$  in the range

$0^\circ < \theta < 360^\circ$  satisfying

$$7 \cos \theta - 24 \sin \theta = 5$$

[6 marks]

(Turn over)

**07 (a)**

The graphs of  $y = 5x - 3$  and

$y = 2x + 3$  intersect at the point  $A$

Show that the coordinates of  $A$  are  
 $(2, 7)$

[2 marks]

**(b)** On the same set of axes, sketch the graphs of

$$y = |5x - 3| \quad \text{and} \quad y = |2x + 3|$$

clearly indicating the coordinates of the points of intersection of the two graphs and the points where the graphs touch the  $X$ -axis. [4 marks]

(Turn over)

**07 (c)**

**Calculate the area of the region satisfying the inequalities**

$$y \geq |5x - 3| \quad \text{and} \quad y \leq |2x + 3|$$

**[4 marks]**

**(Turn over)**

08. The function  $f$  is defined by

$$f(x) = \frac{4x^2 + 12x + 9}{2x^2 + x - 3}$$

where  $x > 1$

(a) Show that  $f(x)$  can be written as

$$2 + \frac{5}{x-1}$$

[3 marks]

(Turn over)

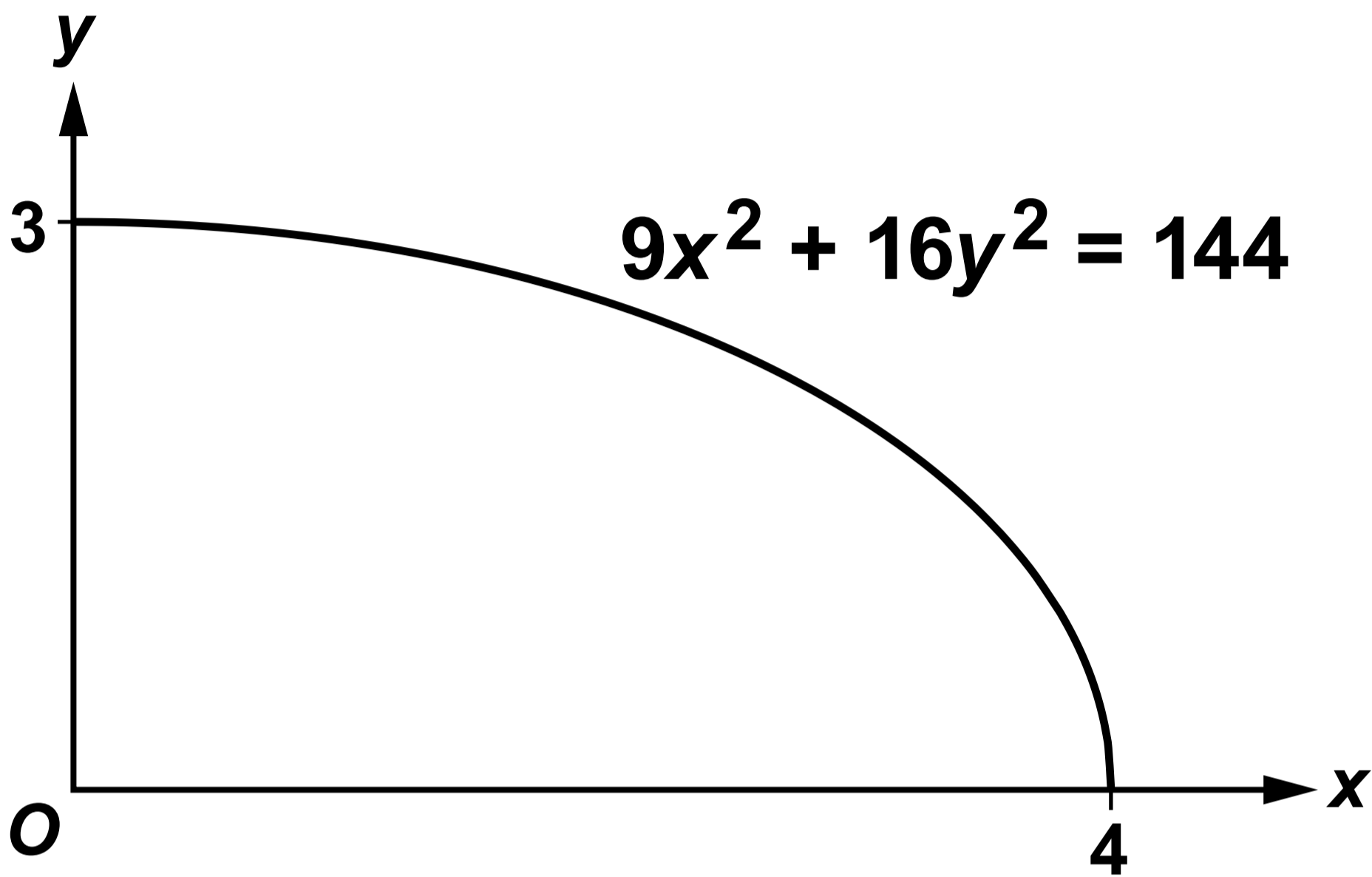
08 (b)

Hence find the exact value of

$$\int_3^7 f(x) dx$$

[4 marks]

(Turn over)



**09. The aerial view of a patio under construction is shown opposite.**

**The curved edge of the patio is described by the equation**

**$9x^2 + 16y^2 = 144$  where  $x$  and  $y$  are measured in metres.**

**To construct the patio, the area enclosed by the curve and the coordinate axes is to be covered with a layer of concrete of depth 0.06 m**

**(Turn over)**

**09 (a)**

**Show that the volume of concrete required for the construction of the patio is given by**

$$0.015 \int_0^4 \sqrt{144 - 9x^2} \, dx$$

**[3 marks]**

**(b) Use the trapezium rule with six ordinates to estimate the volume of concrete required. [4 marks]**

**(c) State whether your answer in part (b) is an overestimate or an underestimate of the volume required. Give a reason for your answer. [1 mark]**

**(Turn over)**

10. Two real functions are defined as

$$f(x) = \frac{8}{x-4} \quad \text{for}$$

$$(-\infty < x < 4) \cup (4 < x < \infty)$$

$$g(x) = (x-2)^2 \quad \text{for } -\infty < x < \infty$$

(Turn over)

10 (a) (i)

Find an expression for  $fg(x)$

[2 marks]

(ii) Determine the values of  $x$  for which  $fg(x)$  does not exist.

[3 marks]

(b) Find an expression for  $f^{-1}(x)$

[3 marks]

(Turn over)

11. A curve  $C$  has equation

$$f(x) = 5x^3 + 2x^2 - 3x$$

(a) Find the  $X$ -coordinate of the point of inflection. State, with a reason, whether the point of inflection is stationary or non-stationary.

[5 marks]

(b) Determine the range of values of  $X$  for which  $C$  is concave. [2 marks]

(Turn over)

**12. The rate of change of a variable  $y$  with respect to  $x$  is directly proportional to  $y$**

**(a) Write down a differential equation satisfied by  $y$  [1 mark]**

**(b) When  $x = 1$  and  $y = 0.5$ , the rate of change of  $y$  with respect to  $x$  is 2  
Find  $y$  when  $x = 3$  [6 marks]**

13. The curve  $C_1$  has parametric equations

$$x = 3p + 1$$

$$y = 9p^2$$

The curve  $C_2$  has parametric equations

$$x = 4q$$

$$y = 2q$$

Find the Cartesian coordinates of the points of intersection of  $C_1$  and  $C_2$

[7 marks]

(Turn over)

14 (a)

Use integration by parts to evaluate

$$\int_0^1 (3x - 1)e^{2x} dx$$

[4 marks]

(b) Use the substitution  $u = 1 - 2 \cos x$  to find

$$\int \frac{\sin x}{1 - 2 \cos x} dx$$

[4 marks]

END OF PAPER