



GCE A LEVEL

1305U40-1

MONDAY, 5 JUNE 2023 – AFTERNOON

FURTHER MATHEMATICS – A2 unit 4

FURTHER PURE MATHEMATICS B

**2 hours 30 minutes plus your additional
time allowance**

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink, black ball-point pen or your usual method.

Answer ALL questions.

Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

(Turn over)

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

ADDITIONAL FORMULAE FOR 2023**LAWS OF LOGARITHMS**

$$\log_a x + \log_a y \equiv \log_a (xy)$$

$$\log_a x - \log_a y \equiv \log_a \left(\frac{x}{y} \right)$$

$$k \log_a x \equiv \log_a (x^k)$$

SEQUENCES

General term of an arithmetic progression:

$$u_n = a + (n - 1) d$$

General term of a geometric progression:

$$u_n = ar^{n-1}$$

(Turn over)

MENSURATION

For a circle of radius, r , where an angle at the centre of θ radians subtends an arc of length S and encloses an associated sector of area A :

$$s = r\theta$$

$$A = \frac{1}{2} r^2 \theta$$

CALCULUS AND DIFFERENTIAL EQUATIONS**DIFFERENTIATION****FUNCTION****DERIVATIVE**

$f(x)g(x)$

$f'(x)g(x) + f(x)g'(x)$

$f(g(x))$

$f'(g(x))g'(x)$

INTEGRATION**FUNCTION****INTEGRAL**

$f'(g(x))g'(x)$

$f(g(x)) + c$

Area under a curve = $\int_a^b y dx$

(Turn over)

REMINDER: Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.

1. The functions f and g have domains $(-1, \infty)$ and $(0, \infty)$ respectively and are defined by

$$f(x) = \cosh x \quad g(x) = x^2 - 1$$

- (a) State the domain and range of fg [2 marks]

- (b) Solve the equation $fg(x) = 3$

Give your answer correct to three decimal places.

[3 marks]

(Turn over)

2. The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} \lambda & 1 & 14 \\ -1 & 2 & 8 \\ -3 & 2 & \lambda \end{pmatrix}$$

where λ is a real constant.

(a) Find an expression for the determinant of **A** in terms of λ

Give your answer in the form $a\lambda^2 + b\lambda + c$, where **a**, **b**, **c** are integers whose values are to be determined.

[3 marks]

(b) Show that **A** is non-singular for all values of λ

[4 marks]

(Turn over)

3 (a)

Given that $z = \cos\theta + i\sin\theta$
use de Moivre's theorem to show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

[3 marks]

(b) Express $32\cos^6\theta$ in the form

$$a\cos 6\theta + b\cos 4\theta + c\cos 2\theta + d$$

where a, b, c, d are integers whose values are to be determined.

[6 marks]

(Turn over)

4. Solve the simultaneous equations

$$4x - 2y + 3z = 8$$

$$2x - 3y + 8z = -1$$

$$2x + 4y - z = 0$$

[5 marks]

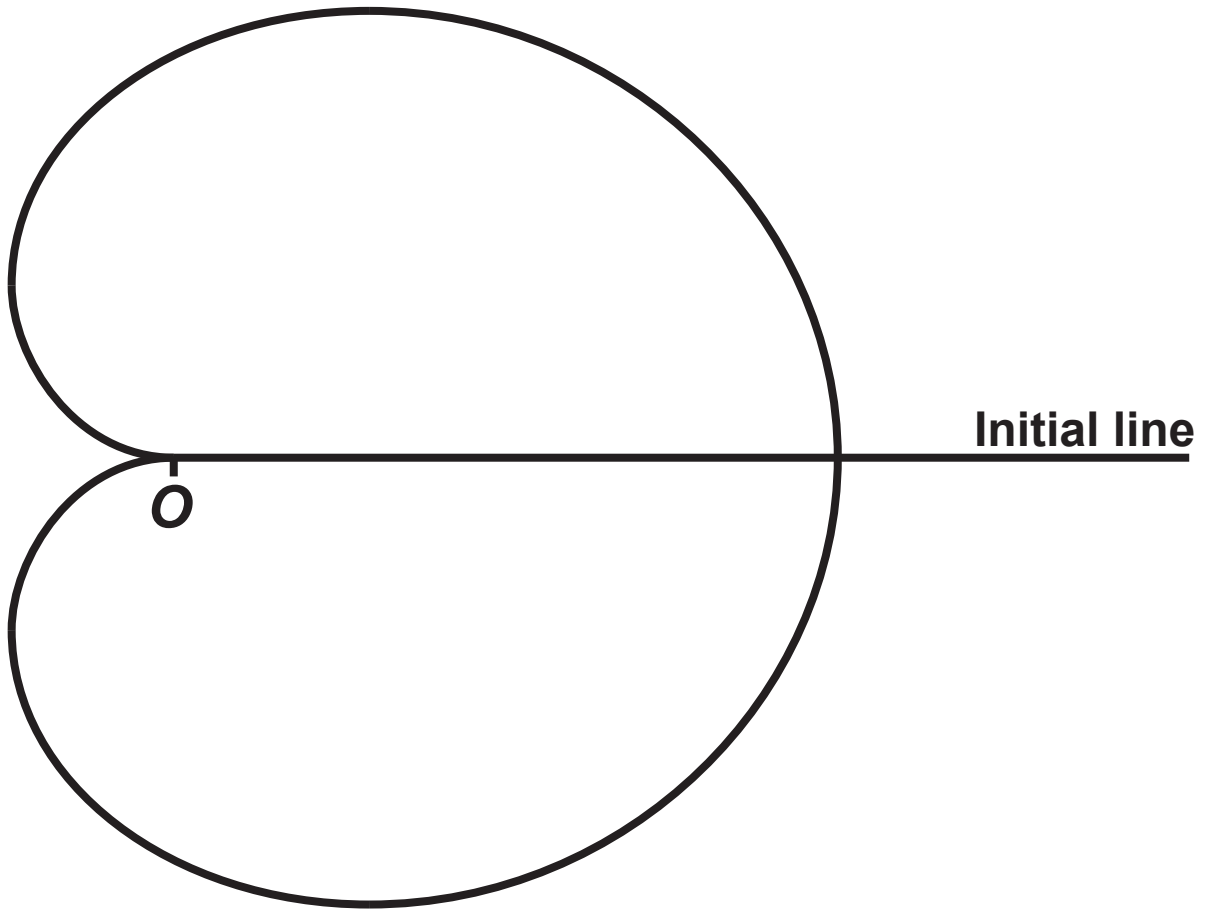
5 (a) Write down and simplify the Maclaurin series for

$\sin 2x$ as far as the term in **x^5** [2 marks]

(b) USING YOUR ANSWER TO PART (a), determine the

Maclaurin series for **$\cos^2 x$** as far as the term
in **x^4** [5 marks]

(Turn over)



6 (a) Show that $\tan \theta$ may be expressed as $\frac{2t}{1-t^2}$

where $t = \tan \left(\frac{\theta}{2} \right)$ [1 mark]

The diagram opposite shows a sketch of the curve C with polar equation

$$r = \cos \left(\frac{\theta}{2} \right) \quad \text{where } -\pi < \theta \leq \pi$$

(b) Show that the θ -coordinate of the points at which the tangent to C is perpendicular to the initial line satisfies the equation

$$\tan \theta = -\frac{1}{2} \tan \left(\frac{\theta}{2} \right)$$

[4 marks]

(Turn over)

6 (c) Hence, find the polar coordinates of the points on C where the tangent is perpendicular to the initial line. [6 marks]

(d) Calculate the area of the region enclosed by the curve C and the initial line for $0 \leq \theta \leq \pi$ [5 marks]

7. Find the cube roots of the complex number $z = 11 - 2i$, giving your answers in the form $x + iy$, where x and y are real and correct to three decimal places. [7 marks]

8. The function f is defined by

$$f(x) = \frac{1}{\sqrt{x^2 + 4x + 3}}$$

(a) Find the mean value of the function f for

$0 \leq x \leq 2$, giving your answer correct to three decimal places. [5 marks]

(b) The region R is bounded by the curve

$y = f(x)$, the x -axis and the lines $x = 0$ and $x = 2$

Find the exact value of the volume of the solid

generated when R is rotated through four right

angles about the x -axis. [6 marks]

9. Consider the differential equation

$$(x + 1) \frac{dy}{dx} + 5y = (x + 1)^2 \quad x > -1$$

Given that $y = \frac{1}{4}$ when $x = 1$

find the value of y when $x = 0$ [8 marks]

10 (a)

By writing $y = \sin^{-1}(2x + 5)$ as

$\sin y = 2x + 5$, show that

$$\frac{dy}{dx} = \frac{2}{\sqrt{1 - (2x + 5)^2}} \quad [5 \text{ marks}]$$

(b) Deduce the range of values of X for which

$$\frac{d}{dx} (\sin^{-1}(2x + 5)) \text{ is valid.} \quad [3 \text{ marks}]$$

11. Evaluate the integrals

(a) $\int_{-2}^0 e^{2x} \sinh x \, dx$ [5 marks]

(b) $\int_{\frac{3}{2}}^3 \frac{5}{(x-1)(x^2+9)} \, dx$ [9 marks]

12. Find the general solution of the equation

$\cos 4\theta + \cos 2\theta = \cos \theta$ [6 marks]

(Turn over)

13. Two species of insects, X and Y , co-exist on an island. The populations of the species at time t years are X and Y respectively, where X and Y are measured in millions. The situation can be modelled by the differential equations

$$\frac{dx}{dt} = 3x + 10y$$

$$\frac{dy}{dt} = x + 6y$$

(a) (i) Show that
$$\frac{d^2x}{dt^2} - 9\frac{dx}{dt} + 8x = 0$$

- (ii) Find the general solution for X in terms of t
[7 marks]

(Turn over)

13 (b)

Find the corresponding general solution for y
[4 marks]

(c) When $t = 0$, $\frac{dx}{dt} = 5$ and the population of

species Y is 4 times the population of
species X

Find the particular solution for X in terms of t

[6 marks]

END OF PAPER