GCE AS MARKING SCHEME

SUMMER 2024

AS
FURTHER MATHEMATICS
UNIT 1 FURTHER PURE MATHEMATICS A
2305U10-1

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE AS FURTHER MATHEMATICS

UNIT 1 FURTHER PURE MATHEMATICS A

SUMMER 2024 MARK SCHEME

Qu	Solution	Mark	Notes
1.	METHOD 1: $\frac{v}{w} = \frac{-16+11\mathrm{i}}{5+2\mathrm{i}} \times \frac{5-2\mathrm{i}}{5-2\mathrm{i}}$	M1	Multiplying by conjugate
	$\frac{v}{w} = \frac{-80+32\mathrm{i}+55\mathrm{i}-22\mathrm{i}^2}{25-10\mathrm{i}+10\mathrm{i}-4\mathrm{i}^2}$		
	$\frac{v}{w} = \frac{-58+87\mathrm{i}}{29} = -2+3\mathrm{i} = z$	A1	No workings M0A0
	$\lvert z\rvert = \sqrt{(-2)^2+3^2} = \sqrt{13}$	A1	FT z
	$\arg z = \tan^{-1}\left(\frac{3}{-2}\right) + 180^{\mathrm{o}} = 123.69^{\mathrm{o}}$	A1	FT z 2.16 radian Accept 124°
	$z = \sqrt{13}(\cos 123.69^{\mathrm{o}} + \mathrm{i}\sin 123.69^{\mathrm{o}})$	B1	FT arg
	METHOD 2: $\lvert v\rvert = \sqrt{(-16)^2+11^2} = \sqrt{377}$ $\lvert w\rvert = \sqrt{5^2+2^2} = \sqrt{29}$	(B1)	Both v and w
	$\arg v = \tan^{-1}\left(\frac{11}{-16}\right) + 180^{\mathrm{o}} = 145.49^{\mathrm{o}}$ $\arg w = \tan^{-1}\left(\frac{2}{5}\right) = 21.80^{\mathrm{o}}$	(B1)	Both v and w 2.54 and 0.38 rad
	$\lvert z\rvert = \frac{\sqrt{377}}{\sqrt{29}} = \sqrt{13}$	(M1)	FT using both mod and arg
	$\arg z = 145.49° - 21.80° = 123.69^{\mathrm{o}}$	(A1)	FT both $\lvert z\rvert$ and $\arg(z)$ correct 2.16 radian Accept 124°
	$z = \sqrt{13}(\cos 123.69^{\mathrm{o}} + \mathrm{i}\sin 123.69^{\mathrm{o}})$	(B1)	FT arg
		Total [5]	

Qu	Solution	Mark	Notes
2.	$x^3 + x^2 - 7x - 15 = (x - 3)(x^2 + 4x + 5)$ When $x - 3 = 0$, $x = 3$	B1	May be awarded at any point
	When $x^2 + 4x + 5 = 0$, Solving, e.g. $x = \frac{-4 \pm \sqrt{4^2 - (4 \times 1 \times 5)}}{2}$ or $(x + 2)^2 + 1 = 0$ $(x + 2)^2 = -1$ $x + 2 = \pm \mathrm{i}$	M1	Complete method M0 no workings
	$x = -2 \pm \mathrm{i}$	A1	Both, cao
		Total [3]	

Qu	Solution	Mark	Notes
3.	METHOD 1:		
	Original quadratic with roots α $\alpha + \alpha = -p$ $\alpha \times \alpha = q$	B1	Both Allow use of other notation for p, q but must be α
	EITHER: New quadratic with roots $\frac{1}{\alpha}$ $\frac{1}{\alpha} + \frac{1}{\alpha} = -m$ $\frac{1}{\alpha} \times \frac{1}{\alpha} = m$	B1	Both si Allow use of other notation for m
	OR: New quadratic with roots $\frac{1}{\alpha}$ $\left(x - \frac{1}{\alpha}\right)\left(x - \frac{1}{\alpha}\right) = 0$ $x^2 - \frac{2}{\alpha}x + \frac{1}{\alpha^2} = 0$	(B1)	
	THEN: $-\left(\frac{1}{\alpha} + \frac{1}{\alpha}\right) = \frac{1}{\alpha} \times \frac{1}{\alpha}$ $\frac{-2}{\alpha} = \frac{1}{\alpha^2}$ $-2\alpha^2 = \alpha$ $2\alpha^2 + \alpha = 0$ $\alpha(2\alpha + 1) = 0$	M1	
	$\therefore \alpha = 0$ or $\alpha = -\frac{1}{2}$	A1	Either value
	As $\alpha \neq 0$, $\alpha = -\frac{1}{2}$.	A1	Must reject $\alpha = 0$
	Therefore, $-\frac{1}{2} + -\frac{1}{2} = -p \quad \rightarrow p = 1$ $-\frac{1}{2} \times -\frac{1}{2} = q \quad \rightarrow q = \frac{1}{4}$	A1	FT their α
	METHOD 2: Let $w = \frac{1}{x}$, then $x = \frac{1}{w}$	(M1)	
	leading to $\left(\frac{1}{w}\right)^2 + p\left(\frac{1}{w}\right) + q = 0$ $qw^2 + pw + 1 = 0.$ $w^2 + \frac{p}{q}w + \frac{1}{q} = 0$	(A1)	
	Hence, $\frac{p}{q} = m$ and $\frac{1}{q} = m$	(A1)	
	therefore $\frac{p/q}{1/q} = \frac{m}{m}$ such that $p = 1$.	(A1)	
	Given than $\frac{1}{\alpha} + \frac{1}{\alpha} = -m$, $-\frac{2}{\alpha} = m \rightarrow \alpha = -\frac{2}{m}$ and $p = -2\alpha = \frac{4}{m} \rightarrow m = 4$	(B1)	
	Therefore, $\frac{1}{q} = 4 \rightarrow q = \frac{1}{4}$	(B1)	

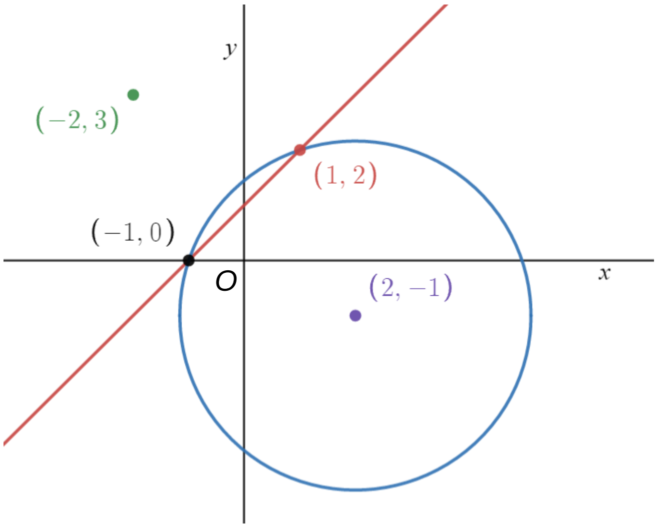
Qu	Solution	Mark	Notes
3.	METHOD 3: Given repeated root, $b^2 - 4ac = 0$ therefore $m^2 - 4m = 0$	(M1)	
	Solving, e.g. $m(m-4) = 0$ $m = 0$ or $m = 4$	(A1)	
	When $m = 0$, no possible roots so new equation is $x^2 + 4x + 4 = 0$	(A1)	Must reject $m = 0$
	Solving, e.g. $(x+2)^2 = 0$ $x = -2$	(A1)	
	Therefore, $-2 = \frac{1}{\alpha} \to \alpha = -\frac{1}{2}$	(B1)	
	Original equation: $\left(x + \frac{1}{2}\right)^2 = 0$ $x^2 + x + \frac{1}{4} = 0$ such that $p = 1, q = \frac{1}{4}$.	(B1)	
		Total [6]	

Qu	Solution	Mark	Notes
4. a)	$u + \mathrm{i}v = \dfrac{x + \mathrm{i}y}{1 - x - \mathrm{i}y}$	M1	
	$u + \mathrm{i}v = \dfrac{x + \mathrm{i}y}{(1 - x) - \mathrm{i}y} \times \dfrac{(1 - x) + \mathrm{i}y}{(1 - x) + \mathrm{i}y}$	m1	
	$u + \mathrm{i}v = \dfrac{(x + \mathrm{i}y)(1 - x) + (x + \mathrm{i}y)(\mathrm{i}y)}{(1 - x)^2 - (\mathrm{i}y)^2}$ $u + \mathrm{i}v = \dfrac{x - x^2 + \mathrm{i}y - \mathrm{i}xy + \mathrm{i}xy - y^2}{(1 - x)^2 + y^2}$ $u + \mathrm{i}v = \dfrac{(x - x^2 - y^2) + \mathrm{i}y}{(1 - x)^2 + y^2}$	A1	
	Comparing coefficients Imaginary parts: $v = \frac{y}{(1-x)^2+y^2}$ (given)	m1	Dependent on M1
	Real parts: $u = \frac{x-x^2-y^2}{(1-x)^2+y^2}$	A1 [5]	Both correct If expanded, denominator must be correct, otherwise A0
b)	Putting $y = 1 - x$	M1	FT their u for method marks only
	$v = \dfrac{y}{y^2 + y^2}\left(= \dfrac{y}{2y^2} = \dfrac{1}{2y}\right)$	A1	
	$u = \dfrac{(1 - y)y - y^2}{y^2 + y^2} = \dfrac{y - 2y^2}{2y^2}$ $u = \dfrac{y}{2y^2} - 1$	A1	
	Eliminating y, the equation of the locus Q is	m1	
	$u = v - 1$	A1	cao oe
	OR Putting $y = 1 - x$	(M1)	FT their u for method marks only
	$v = \dfrac{1 - x}{(1 - x)^2 + (1 - x)^2}\left(= \dfrac{1 - x}{2(1 - x)^2} = \dfrac{1}{2(1 - x)}\right)$	(A1)	
	$u = \dfrac{x - x^2 - (1 - x)^2}{(1 - x)^2 + (1 - x)^2}$ $u = \dfrac{x - x^2 - 1 + 2x - x^2}{2(1 - x)^2} = \dfrac{-2x^2 + 3x - 1}{2(1 - x)^2}$ $u = \dfrac{(1 - x)(2x - 1)}{2(1 - x)^2} = \dfrac{2x - 1}{2(1 - x)} = \dfrac{1 - 2(1 - x)}{2(1 - x)}$ $u = \dfrac{1}{2(1 - x)} - 1$	(A1)	
	Eliminating x, the equation of the locus Q is	(m1)	
	$u = v - 1$	(A1) [5] **Total [10]**	cao oe

Qu	Solution	Mark	Notes
5.	METHOD 1:		
	$\sum_{r=k}^{76}(r-31)=\sum_{r=1}^{76}(r-31)-\sum_{r=1}^{k-1}(r-31)$	M1	Condone $\sum_{r=1}^{76} ⬚ - \sum_{r=1}^{k} ⬚$
	$=\left[\left(\frac{1}{2}\times 76\times 77\right)-(31\times 76)\right]$ $-\left[\left(\frac{1}{2}\times(k-1)\times k\right)-\left(31\times(k-1)\right)\right]$	m1 A1	Use of $\sum ⬚$ formulae All correct
	METHOD 2:		
	$\sum_{r=k}^{76}(r-31)=\sum_{r=1}^{76}(r-31)-\sum_{r=1}^{k-1}(r-31)$	(M1)	Condone $\sum_{r=1}^{76} ⬚ - \sum_{r=1}^{k} ⬚$
	$\sum_{r=1}^{n}(r-31)=\sum_{r=1}^{n}r-\sum_{r=1}^{n}31$ $=\frac{n(n+1)}{2}-31n$ $=\frac{n(n-61)}{2}$	(m1)	Use of $\sum ⬚$ formulae
	$\sum_{r=k}^{76}(r-31)=\frac{76\times 15}{2}-\frac{(k-1)(k-62)}{2}$	(A1)	All correct
	THEN		
	If $\sum_{r=k}^{76}(r-31)=980$,		
	$570-\left[\left(\frac{1}{2}\times(k-1)\times k\right)-\left(31\times(k-1)\right)\right]=980$	M1	FT their expression provided at least quadratic
	$570-\frac{k^2}{2}+\frac{63k}{2}-31=980$		
	$\frac{k^2}{2}-\frac{63k}{2}+441=0$	A1	cao, oe
	Solving, $k^2-63k+882=0$	m1	m0 no working FT their quadratic for m1
	$(k-21)(k-42)=0$		
	$k=21$ or $k=42$ $\therefore k$ has two possible values.	A1	cao
	If m0A0, then SC1 for:		
	Discriminant = $(-63)^2-(4\times 1\times 882)$ Discriminant = $441>0$ $\therefore k$ has two possible values		
	Note: Trial and improvement, 0 marks		
		Total [7]	

Qu	Solution	Mark	Notes
6. a)	METHOD 1:		
	L_1		
	$\|z-2+\mathrm{i}\| = \|z+2-3\mathrm{i}\|$		
	$\|x+\mathrm{i}y-2+\mathrm{i}\| = \|x+\mathrm{i}y+2-3\mathrm{i}\|$	M1	si
	$\|(x-2)+\mathrm{i}(y+1)\| = \|(x+2)+\mathrm{i}(y-3)\|$	A1	si
	$(x-2)^2+(y+1)^2 = (x+2)^2+(y-3)^2$	m1	
	$x^2-4x+4+y^2+2y+1 = x^2+4x+4+y^2-6y+9$		
	$-8x+8y-8=0$	A1	oe
	$y=x+1$		
	L_2		
	$\|z-2+\mathrm{i}\| = \sqrt{10}$		
	$\|x+\mathrm{i}y-2+\mathrm{i}\| = \sqrt{10}$		Award M1A1m1 here if not awarded for L_1
	$\|(x-2)+\mathrm{i}(y+1)\| = \sqrt{10}$		
	$(x-2)^2+(y+1)^2 = 10$	A1	
	$x^2-4x+4+y^2+2y+1 = 10$		
	$x^2+y^2-4x+2y-5=0$		
	Substituting from L_1 into L_2:	m1	FT their equation of a line
	$x^2+(x+1)^2-4x+2(x+1)-5=0$		
	$2x^2-2=0$	m1	Solving equation
	$x^2=1$		
	$\therefore\ x=1 \quad \text{or} \quad x=-1$	A1	cao
	OR		
	$(y-1)^2+y^2-4(y-1)+2y-5=0$		
	$2y^2-4y=0$		
	$2y(y-2)=0$		Award m1A1 for y if not awarded for x
	$\therefore\ y=0 \quad \text{or} \quad y=2$		
	Therefore points of intersection are $(1,2)$ and $(-1,0)$	B1	Must be paired correctly FT their x or y into their equation of a line provided real

Qu	Solution	Mark	Notes
6. a)	METHOD 2: L_2 $\lvert z-2+\mathrm{i}\rvert=\sqrt{10}$ $\lvert x+\mathrm{i}y-2+\mathrm{i}\rvert=\sqrt{10}$ $\lvert(x-2)+\mathrm{i}(y+1)\rvert=\sqrt{10}$ $(x-2)^2+(y+1)^2=10$ $x^2-4x+4+y^2+2y+1=10$ $x^2+y^2-4x+2y-5=0$	(M1) (A1) (m1) (A1)	si si
	Equating L_1 to $\sqrt{10}$: $\lvert z+2-3\mathrm{i}\rvert=\sqrt{10}$ $\lvert x+\mathrm{i}y+2-3\mathrm{i}\rvert=\sqrt{10}$ $\lvert(x+2)+\mathrm{i}(y-3)\rvert=\sqrt{10}$ $(x+2)^2+(y-3)^2=10$ $x^2+4x+4+y^2-6y+9=10$ $x^2+y^2+4x-6y+3=0$	(A1)	Award M1A1m1 here if not awarded for L_2
	Finding the equation of L_1 and substituting into equation of circle: $(x^2+y^2+4x-6y+3)-(x^2+y^2-4x+2y-5)=0-0$ $8x-8y+8=0$ $y=x+1$	(m1)	oe
	$x^2+(x+1)^2-4x+2(x+1)-5=0$ $2x^2-2=0$ $\therefore\ x=1$ or $x=-1$ OR $(y-1)^2+y^2-4(y-1)+2y-5=0$ $2y^2-4y=0$ $2y(y-2)=0$ $\therefore\ y=0$ or $y=2$ OR $x^2+(x+1)^2+4x-6(x+1)+3=0$ $2x^2-2=0$ $\therefore\ x=1$ or $x=-1$ OR $(y-1)^2+y^2+4(y-1)-6y+3=0$ $2y^2-4y=0$ $2y(y-2)=0$ $\therefore\ y=0$ or $y=2$	(m1) (A1)	Solving to find x or y Award m1A1 for any of these 4 options of solutions
	Therefore points of intersection are $(1,2)$ and $(-1,0)$	(B1)	Must be paired correctly FT their x or y into their equation of a line provided real

Qu	Solution	Mark	Notes
6. a)	METHOD 3: (Geometric argument) L_1 is perp bisector of line of (2, -1) and (-2, 3) $m = \frac{3--1}{-2-2} = \frac{4}{-4} = -1 \rightarrow m(\text{perp}) = 1$ Midpoint: (0,1) Equation of L_1: $y = x + 1$ L_2 is a circle centred at (2, -1) with radius $\sqrt{10}$, such that $(x-2)^2 + (y+1)^2 = 10$ leading to $(x-2)^2 + (x+2)^2 = 10$ $2x^2 + 8 = 10$ $x^2 = 1$ $\therefore x = \pm 1$ When $x = 1, y = 2$ and when $x = -1, y = 0$	M1 A1 A1 m1 A1 m1 A1 B1 B1 [9]	 oe or $y = 2, 0$ FT their x provided real If B0B0, award SC1 for incorrectly paired coordinates
b)		G1 G1 G1 [3] **Total [12]**	Circle, centred in 4th quadrant, extending into all 3 other quads Line, positive gradient, intersecting circle twice FT their line Fully correct, with points of intersection clearly labelled

Qu	Solution	Mark	Notes
7.	When $n = 1$, $13^1 + 8 = 21$ which is a multiple of 7. Therefore, proposition is true for $n = 1$.	B1	
	Assume the proposition is true for $n = k$	M1	
	i.e. $13^{2k-1} + 8$ is a multiple of 7 or $13^{2k-1} + 8 = 7N$		
	Consider $n = k + 1$		
	$13^{2(k+1)-1} + 8 = 13^{2k-1+2} + 8$	M1	
	$= 13^2(13^{2k-1}) + 8$	A1	
	$= 169(7N - 8) + 8$	A1	$169(13^{2k-1} + 8) - 1344$
	$= 1183N - 1344$		
	$= 7(169N - 192)$		
	Since this is a multiple of 7, $13^{2(k+1)-1} + 8$ is also a multiple of 7.	A1	
	So, if the proposition is true for $n = k$, it is also true for $n = k + 1$. Since we have shown it is true for $n = 1$, by mathematical induction, it is true for all positive integers n.	A1	cso
		Total [7]	

Qu	Solution	Mark	Notes
8. a)	METHOD 1: When $\cos\theta = 0.8$ and θ acute, $\sin\theta = 0.6$		
	Rotation matrix: $\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}$	B1	si
	$\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}[\text{Reflection}] = \frac{1}{85}\begin{bmatrix} -84 & -13 \\ -13 & 84 \end{bmatrix}$	M1	Attempt to use rotation matrix in equation
	$[\text{Reflection}] = \frac{1}{85}\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}^{-1}\begin{bmatrix} -84 & -13 \\ -13 & 84 \end{bmatrix}$	m1	Use of inverse
	$\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}^{-1} = \frac{1}{1}\begin{bmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{bmatrix}$	A1	
	$[\text{Reflection}] = \frac{1}{85}\begin{bmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{bmatrix}\begin{bmatrix} -84 & -13 \\ -13 & 84 \end{bmatrix}$	m1	Multiplication
	$[\text{Reflection}] = \begin{bmatrix} -\frac{15}{17} & \frac{8}{17} \\ \frac{8}{17} & \frac{15}{17} \end{bmatrix}$	A1	cao
	As $[\text{Reflection}] = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix}$ $\cos 2\alpha = -\frac{15}{17}$ AND $\sin 2\alpha = \frac{8}{17}$ (2α is in the second quadrant)	B1	FT their reflection matrix provided of correct format
	$2\alpha = 151.9(275131\ldots)$ $\alpha = 75.9(6375653\ldots)$	B1	
	$y = (\tan 75.96375653\ldots)x = 4x$ Therefore, $k = 4$	B1	

Qu	Solution	Mark	Notes
8. a)	METHOD 2: When $\cos\theta = 0.8$ and θ acute, $\sin\theta = 0.6$		
	Rotation matrix: $\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}$	(B1)	si
	Let reflection matrix be $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{85}\begin{bmatrix} -84 & -13 \\ -13 & 84 \end{bmatrix}$	(M1)	
	$0.8a - 0.6c = -\frac{84}{85}$ and $0.6a + 0.8c = -\frac{13}{85}$	(m1)	Attempt to form simultaneous eqn
	$0.8b - 0.6d = -\frac{13}{85}$ and $0.6b + 0.8d = \frac{84}{85}$	(A1)	All correct
	Solving simultaneous equations	(m1)	m0A0 no working
	$a = -\frac{15}{17}$ $b = \frac{8}{17}$ $c = \frac{8}{17}$ $d = \frac{15}{17}$	(A1)	cao
	$[\text{Reflection}] = \begin{bmatrix} -\frac{15}{17} & \frac{8}{17} \\ \frac{8}{17} & \frac{15}{17} \end{bmatrix}$		
	As $[\text{Reflection}] = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix}$ $\cos 2\alpha = -\frac{15}{17}$ AND $\sin 2\alpha = \frac{8}{17}$ (2α is in the second quadrant)	(B1)	FT their reflection matrix provided of correct format
	$2\alpha = 151.9(275131\ldots)$ $\alpha = 75.9(6375653\ldots)$	(B1)	
	$y = (\tan 75.96375653\ldots)x = 4x$ Therefore, $k = 4$	(B1)	

Qu	Solution	Mark	Notes
8. a)	METHOD 3: When $\cos\theta = 0.8$ and θ acute, $\sin\theta = 0.6$ Rotation matrix: $\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}$	(B1)	si
	Let reflection matrix be $\begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix}$ $\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}\begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix} = \frac{1}{85}\begin{bmatrix} -84 & -13 \\ -13 & 84 \end{bmatrix}$	(M1)	
	$0.8\cos 2\alpha - 0.6\sin 2\alpha = -\frac{84}{85}$ and $0.6\cos 2\alpha + 0.8\sin 2\alpha = -\frac{13}{85}$	(m1) (A1)	Attempt to form simultaneous eqn All correct
	Solving $\cos 2\alpha = -\frac{15}{17}$	(m1) (A1)	m0A0 no working cao Accept $\sin 2\alpha = \frac{8}{17}$
	As $[\text{Reflection}] = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix}$ $\cos 2\alpha = -\frac{15}{17}$ AND $\sin 2\alpha = \frac{8}{17}$ (2α is in the second quadrant)	(B1)	FT their $\cos 2\alpha$ or $\sin 2\alpha$
	$2\alpha = 151.9(275131\ldots)$ $\alpha = 75.9(6375653\ldots)$	(B1)	
	$y = (\tan 75.96375653\ldots)x = 4x$ Therefore, $k = 4$	(B1) [9]	
8. b)	$T = \frac{1}{85}\begin{pmatrix} -84 & -13 \\ -13 & 84 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$	M1	
	Giving, $\frac{1}{85}(-84x - 13y) = x$ or $\frac{1}{85}(-13x + 84y) = y$	A1	
	Leading to $169x + 13y = 0$ or $13x + y = 0$ Therefore the line of fixed points is $13x + y = 0$.	A1 [3] **Total [12]**	

Qu	Solution	Mark	Notes
9. a)	$\Pi_1: \mathbf{r}.(4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = 5$, let $\mathbf{n}_1 = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ $\Pi_2: \mathbf{r}.(6\mathbf{i} + \mathbf{j} + \mathbf{k}) = 9$, let $\mathbf{n}_2 = 6\mathbf{i} + \mathbf{j} + \mathbf{k}$ $\mathbf{n}_1.\mathbf{n}_2 = \lvert\mathbf{n}_1\rvert\lvert\mathbf{n}_2\rvert\cos\theta$ $\cos\theta = \frac{(4\times 6)+(-3\times 1)+(2\times 1)}{\sqrt{4^2+(-3)^2+2^2}\sqrt{6^2+1^2+1^2}}$ $\cos\theta = \frac{23}{\sqrt{29}\sqrt{38}}$ $\theta = 46.1^{\text{o}}$	B1 M1 A1 A1 [4]	Both $\mathbf{n}_1, \mathbf{n}_2$ si Use of Mark final answer Accept 0.805 rad
b)	$D = \frac{\lvert(4\times 5)+(-3\times -2)+(2\times -6)-5\rvert}{\sqrt{4^2+(-3)^2+2^2}}$ $D = \frac{9}{\sqrt{29}} \quad (1.67\ldots)$	M1 A1 [2]	
c) i)	Point *B*: $(4\times 5)+(-3\times 5)+(2\times 0) = 5$ Point *C*: $(6\times 1)+(1\times 3)+(1\times 0) = 9$	 B1	 Both shown convincingly
ii)	Any valid plane equation e.g. $z = 0$ (because z-coordinate in *B*, *C* is 0) $-x + 2y + kz = 5$ (where $k \in R$)	B1 [2] **Total [8]**	