GCE A LEVEL

1300U30-1

TUESDAY, 7 JUNE 2022 – AFTERNOON

MATHEMATICS – A2 unit 3

PURE MATHEMATICS B

2 hours 30 minutes plus your additional time allowance

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- **a WJEC pink 16-page answer booklet;**
- **a Formula Booklet;**
- **a calculator.**

INSTRUCTIONS TO CANDIDATES

Use black ink, black ball-point pen or your usual method. You may use a pencil for graphs and diagrams only.

Answer ALL questions.

Write your answers in the separate answer booklet provided, following the instructions on the front of the answer booklet.

(Turn over)

Use both sides of the paper. Please only write within the white areas of the booklet.

Write the question number in the left hand margin at the start of each answer. Write the sub parts, e.g. (a), (b) and (c), within the white areas of the booklet.

Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

(Turn over)

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

(Turn over)

REMINDER: Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.

01. Solve the equation

$$6\sec^2 x - 8 = \tan x$$

for $0° \leqslant x \leqslant 360°$

[6 marks]

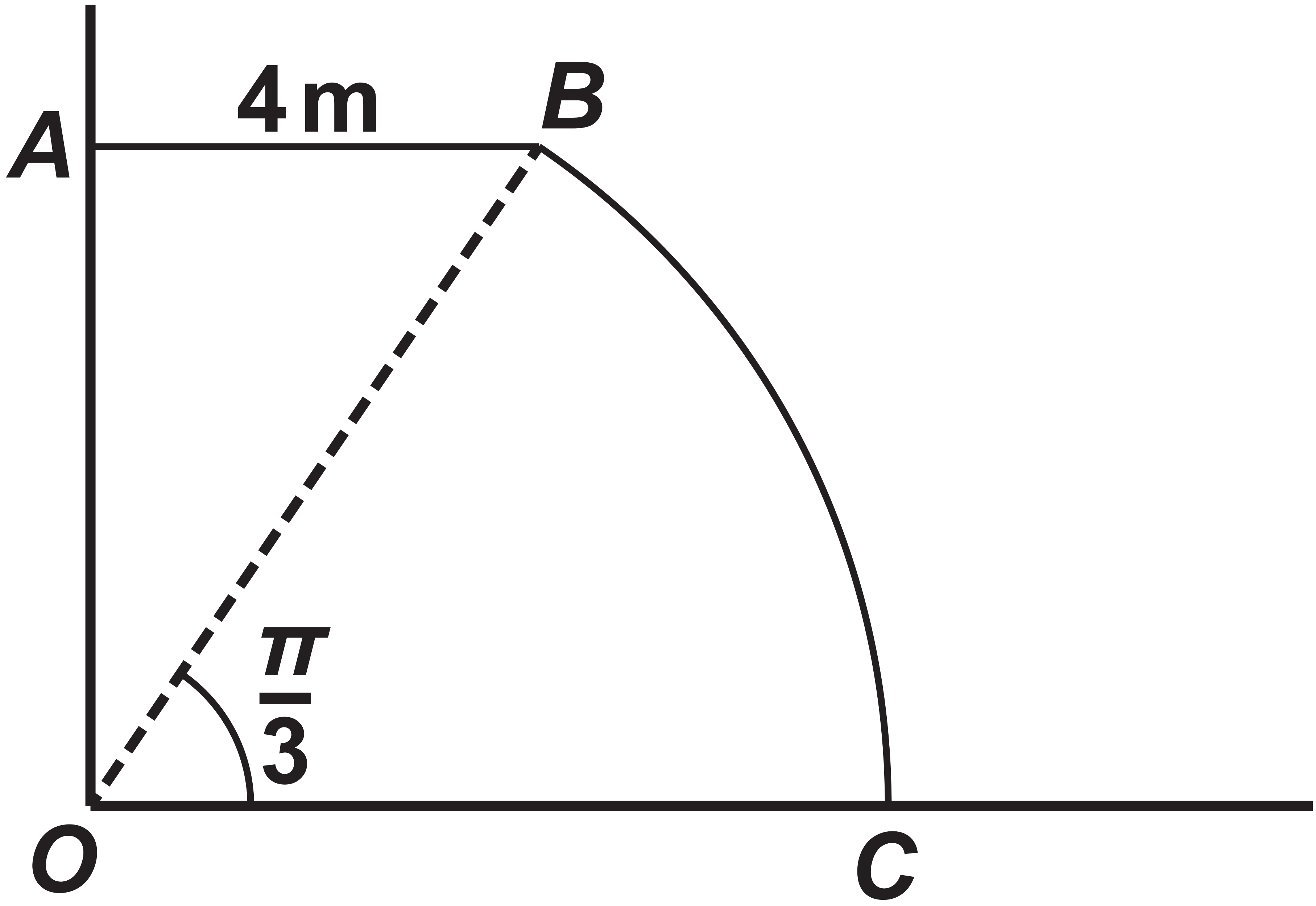
(Turn over)

02. **Differentiate the following functions with respect to x**

(a) $x^3 \ln(5x)$ **[3 marks]**

(b) $(x + \cos 3x)^4$ **[2 marks]**

(Turn over)


A
4 m
B
π
3
O
C

03. **The diagram opposite shows a plan of the patio Eric wants to build.**

The walls OA and OC are perpendicular. The straight line AB is of length 4 m and is perpendicular to OA
The shape OBC is a sector of a circle with centre O and radius OC

The angle BOC is $\frac{\pi}{3}$ radians.

Calculate the area of the patio $OABC$
Give your answer correct to 2 decimal places.

[4 marks]

(Turn over)

04. The sum to infinity of a geometric series with first term a and common ratio r is 120
The sum to infinity of another geometric series with first term a and common ratio $4r^2$ is $112\frac{1}{2}$

Find the possible values of r and the corresponding values of a

[6 marks]

(Turn over)

05. **The function $f(x)$ is defined by**

$$f(x) = \frac{6x + 4}{(x - 1)(x + 1)(2x + 3)}$$

(a) **Express $f(x)$ in terms of partial fractions.** **[4 marks]**

(Turn over)

05. (b)

Find

$$\int \frac{3x + 2}{(x - 1)(x + 1)(2x + 3)}\, dx$$

giving your answer in the form

$a\ln|g(x)|$ where a is a real number

and $g(x)$ is a function of x

[5 marks]

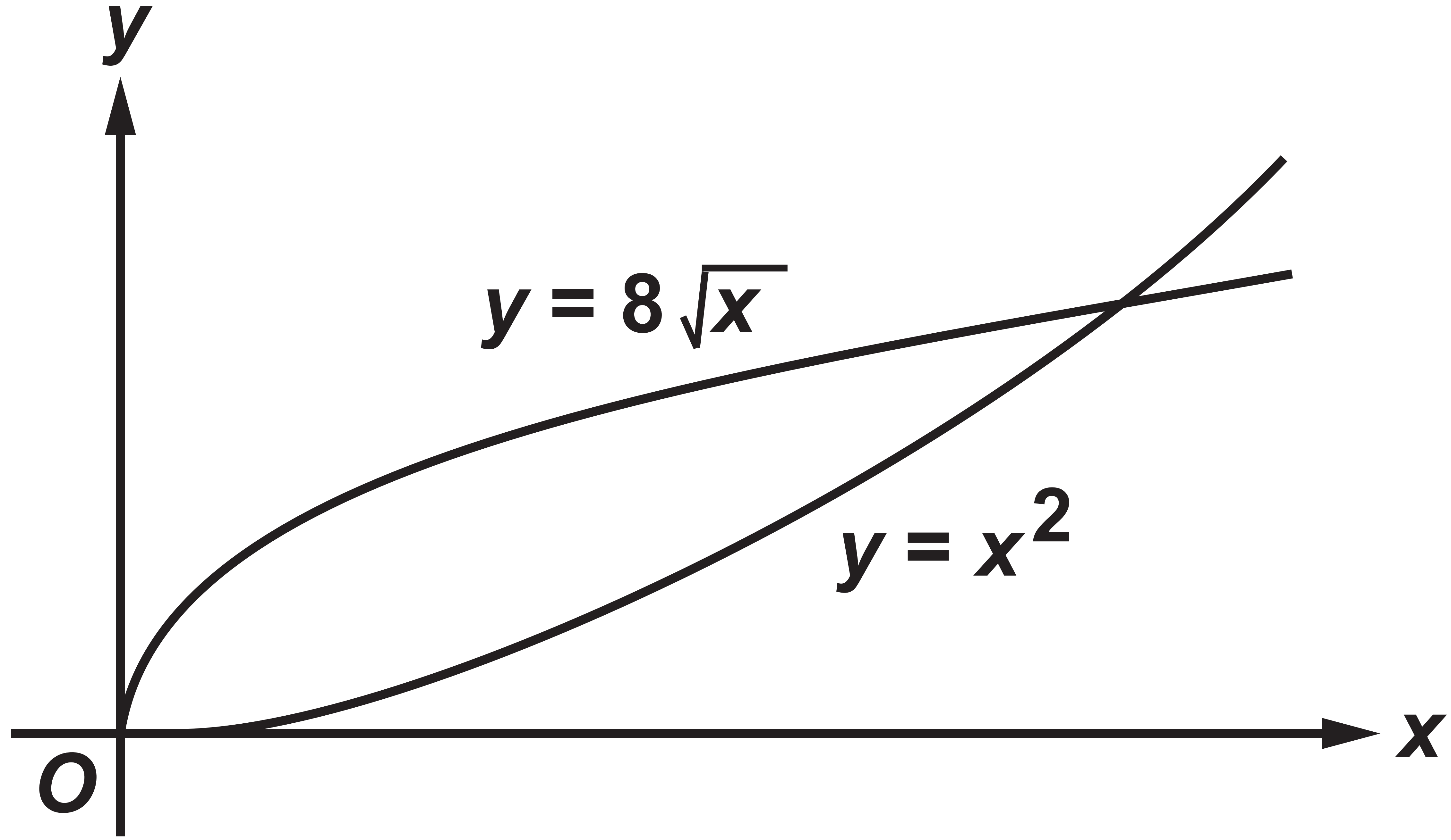
(Turn over)

06. **Geraint opens a savings account. He deposits £10 in the first month. In each subsequent month, the amount he deposits is 20 pence greater than the amount he deposited in the previous month.**

(a) **Find the amount that Geraint deposits into the savings account in the 12th month.** **[2 marks]**

(b) **Determine the number of months it takes for the total amount in the savings account to reach £954.** **[3 marks]**

(Turn over)


y
y = 8 √x
y = x2
x
O

07. **The diagram opposite shows a sketch of the curves $y = x^2$ and $y = 8\sqrt{x}$**

Find the area of the region bounded by the two curves. **[7 marks]**

(Turn over)

08. **Find the first three terms in the binomial expansion of**

$$\frac{2 - x}{\sqrt{1 + 3x}}$$

in ascending powers of x

State the range of values of x for which the expansion is valid.

By writing $x = \frac{1}{22}$ in your expansion, find an approximate value for $\sqrt{22}$ in the form $\frac{a}{b}$ where a, b are integers whose values are to be found. **[8 marks]**

(Turn over)

09. For each of the following sequences, find the first 5 terms, u_1 to u_5

Describe the behaviour of each sequence.

(a) $u_n = \sin\left(\dfrac{n\pi}{2}\right)$

[2 marks]

(b) $u_6 = 33$

$u_n = 2u_{n-1} - 1$

[3 marks]

(Turn over)

10. **Solve the equation**

$$\frac{6x^5 - 17x^4 - 5x^3 + 6x^2}{(3x + 2)} = 0$$

[5 marks]

(Turn over)

11. (a) Express $9\cos x + 40\sin x$ in the form $R\cos(x - \alpha)$ where R and α are constants with $R > 0$ and $0^\circ < \alpha < 90^\circ$

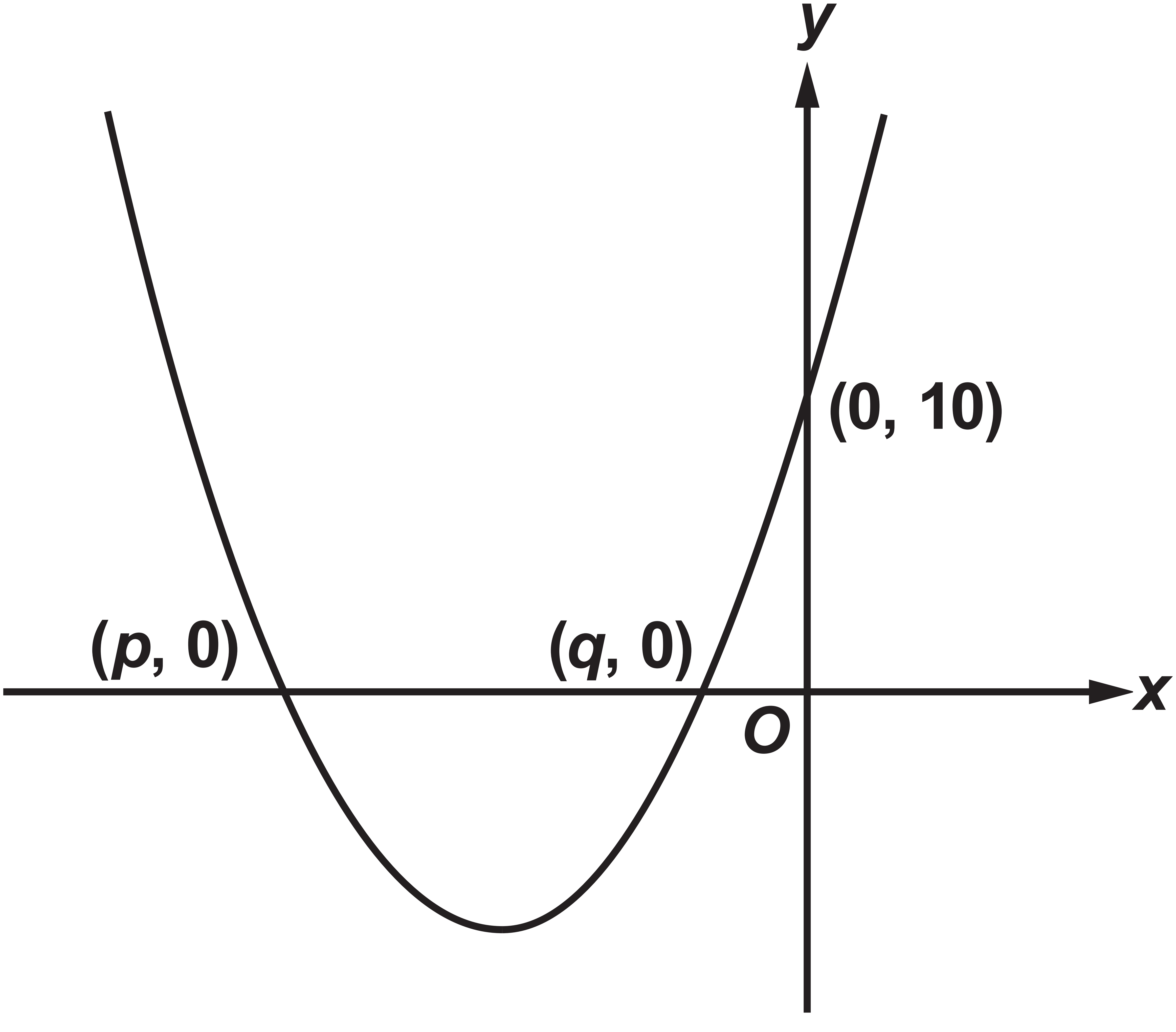
[3 marks]

(b) Find the maximum possible value of

$$\frac{12}{9\cos x + 40\sin x + 47}$$

[2 marks]

(Turn over)


y
(0, 10)
(p, 0)
(q, 0)
O
x

12. **The diagram opposite shows a sketch of the graph of $y = f(x)$**

where $f(x) = 2x^2 + 12x + 10$

The graph intersects the x-axis at the points $(p, 0)$, $(q, 0)$ and the y-axis at the point $(0, 10)$

(a) **Write down the value of $ff(p)$**

[1 mark]

(Turn over)

12. (b)

Determine the values of p and q

[2 marks]

(c) Express $f(x)$ in the form $a(x + b)^2 + c$ where a, b, c are constants whose values are to be found. Write down the coordinates of the minimum point. [3 marks]

(d)

Explain why $f^{-1}(x)$ does not exist. [1 mark]

(Turn over)

12. (e)

The function $g(x)$ is defined as

$$g(x) = f(x) \quad \text{for} \quad -3 \leqslant x < \infty$$

(i) Find an expression for $g^{-1}(x)$

[4 marks]

(ii) Sketch the graph of $y = g^{-1}(x)$ indicating the coordinates of the points where the graph intersects the x-axis and the y-axis.

[2 marks]

(Turn over)

13. **A function is defined by**

$$f(x) = 2x^3 + 3x - 5$$

(a) **Prove that the graph of $f(x)$ does not have a stationary point.**

[2 marks]

(b) **Show that the graph of $f(x)$ does have a point of inflection and find the coordinates of the point of inflection.**

[4 marks]

(c) **Sketch the graph of $f(x)$** **[1 mark]**

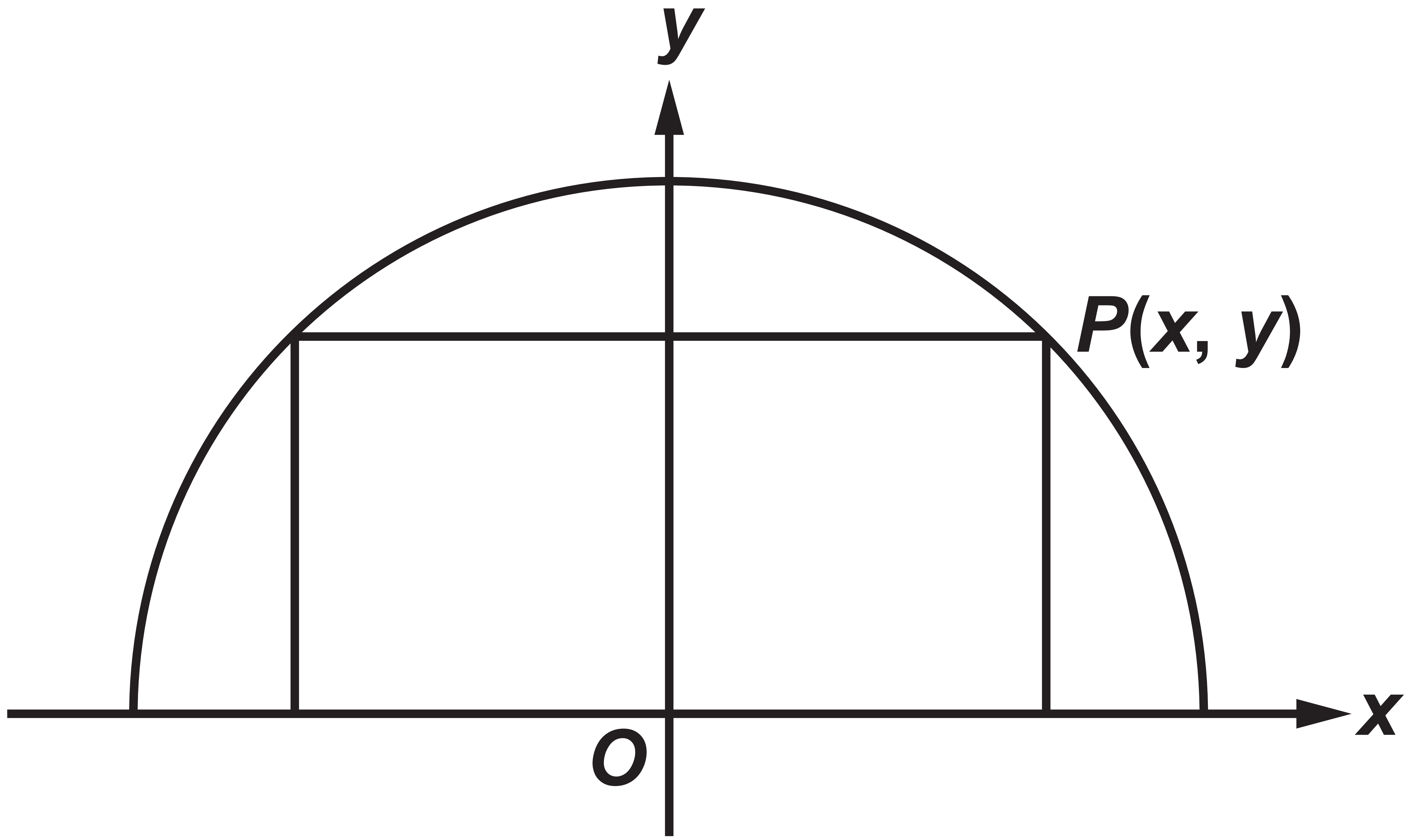
(Turn over)

14. **Evaluate the integral**

$$\int_0^{\pi} x^2 \sin x \, dx$$

[6 marks]

(Turn over)


y
P(x, y)
x
O

15. **A rectangle is inscribed in a semicircle with centre O and radius 4**

The point $P(x, y)$ is the vertex of the rectangle in the first quadrant as shown in the diagram opposite.

(a) **Express the area A of the rectangle as a function of x** **[2 marks]**

(b) **Show that the maximum value of A occurs when $y = x$** **[7 marks]**

(Turn over)

16. The parametric equations of the curve C are

$$x = 3 - 4t + t^2 \qquad\qquad y = (4 - t)^2$$

(a) Find the coordinates of the points where C meets the y-axis. [3 marks]

(b) Show that the x-axis is a tangent to the curve C [5 marks]

(Turn over)

$$\cos(\alpha - \beta) + \sin(\alpha + \beta) \equiv (\cos\alpha + \sin\alpha)(\cos\beta + \sin\beta)$$

17. (a)

Prove the identity opposite.

[2 marks]

(b) (i)

Hence show that

$$\frac{\cos 3\theta + \sin 5\theta}{\cos 4\theta + \sin 4\theta}$$

can be expressed as $\cos\theta + \sin\theta$

(Turn over)

17. (b) (ii)

Explain why

$$\frac{\cos 3\theta + \sin 5\theta}{\cos 4\theta + \sin 4\theta} \neq \cos\theta + \sin\theta$$

when $\theta = \dfrac{3\pi}{16}$ **[3 marks]**

(Turn over)

18. (a)

Use a suitable substitution to find

$$\int \frac{x^2}{(x + 3)^4}\, dx$$

[5 marks]

(b) Hence evaluate

$$\int_0^1 \frac{x^2}{(x + 3)^4}\, dx$$

[2 marks]

END OF PAPER